Calculus II
Final Exam

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2 May 2002

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).*
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.

Name _____

Section _____

Question	Score	Total
1		20
2		10
3		10
4		15
5		15
6		10
7		10
8		15
min(Total,100)		100

1. Compute 4 of the 5 indefinite integrals below. Write here _____ the letter of the integral that is not to be graded. If you do not specify the an integral which is not to be graded, we will take the four lowest scores.

   (a) $\displaystyle\int \frac{x}{x^2+4}\,dx$

   (b) $\displaystyle\int \frac{1}{(9-x^2)^{3/2}}\,dx$

   (c) $\displaystyle\int \frac{x}{x^2+6x+10}\,dx$

   (d) $\displaystyle\int x\sin(3x)\,dx$

   (e) $\displaystyle\int \frac{1}{2+\sqrt{x}}\,dx$

2. Find the values of λ for which $y(x) = e^{\lambda x}$ satisfies the equation

$$y'' - 4y' - 5y = 0.$$

3. Find the sum of each of the following series.

(a) $\dfrac{1}{4 \cdot 3} + \dfrac{1}{4 \cdot 9} + \dfrac{1}{4 \cdot 27} + \dfrac{1}{4 \cdot 81} + \ldots + \dfrac{1}{4 \cdot 3^n} + \ldots$

(b) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{3}{n} - \frac{3}{n+1} \right)$

4. (a) Write the MacLaurin series or Taylor series about 0 for e^x.

   (b) Use your answer in part a) to find the MacLaurin series for e^{-x^3}.

   (c) Find the MacLaurin series for

$$\int_0^x e^{-t^3}\, dt.$$

5. The trapezoid rule T_n and Simpson's rule S_n for approximating the integral $\int_a^b f(x)\,dx$ are

$$\begin{aligned} T_n &= \frac{h}{2}(f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)) \\ S_n &= \frac{h}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n)) \end{aligned}$$

The errors satisfy

$$|E_T| \leq \frac{M_2(b-a)^3}{12n^2} \quad \text{and} \quad |E_S| \leq \frac{M_4(b-a)^5}{180n^4}$$

where M_k is a number which satisfies $|f^{(k)}(x)| \leq M_k$ for all x with $a \leq x \leq b$.

(a) Use the trapezoid rule and Simpson's rule with $n = 4$ to approximate the integral

$$\int_4^7 \cos(3x)\,dx.$$

Give your answers correctly rounded to four decimal places.

(b) Find n so that the error in the trapezoid rule is at most 10^{-4},

$$\left| \int_4^7 \cos(3x)\,dx - T_n \right| \leq 10^{-4}.$$

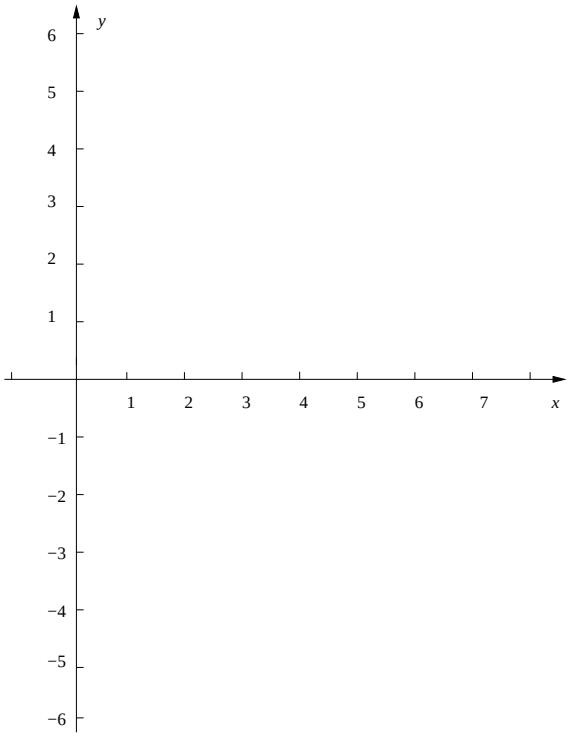
(c) Find n so that the error in Simpson's rule is at most 10^{-4},

$$\left| \int_4^7 \cos(3x)\,dx - S_n \right| \leq 10^{-4}.$$

6. (a) Sketch the parametric curve defined by $x(t) = 2t^2$, $y(t) = t^3 - t$, for $-2 < t < 2$.

   (b) Find the values of t so that $(x, y) = (2, 0)$.

   (c) Find all tangent lines to this curve at $(x, y) = (2, 0)$.



7. (a) Write the length of the curve $x(t) = t^2$, and $y(t) = t^3$ for $0 \leq t \leq 1$ as an integral.

   (b) Evaluate the integral in part a).

8. (a) Sketch the curve defined in polar coordinates by $r = 2\cos(3\theta)$.

   (b) Give an interval of θ which corresponds to one leaf of this curve.

   (c) Find the area enclosed by one leaf of this rose.