Calculus II
Exam 4

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Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).*
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.

Name ______________________

Section ___________

Question	Score	Total
1		5
2		12
3		10
4		15
5		5
6		10
7		15
8		15
9		10
10		10
min(Total,100)		100

1. If $\int_2^4 f(x)\,dx = 2$, find $\int_1^2 f(2x)\,dx$.

2. Let $f(x) = x^3 + 2x + 5$

   (a) Use calculus to show that f is one to one.

   (b) Let g be the inverse function, f^{-1} and find $g(5)$.

   (c) Find the derivative, $g'(5)$.

3. Suppose that the number of critters in a population is $P(t)$ after t days. We assume that the population grows exponentially, $P(2) = 100$ and $P(5) = 350$.

   (a) Find an expression for $P(t)$.

   (b) Find the initial population $P(0)$.

   (c) Find the value of k in the differential equation, $P'(t) = kP(t)$, which P satisfies.

4. Use calculus to evaluate the integrals.

   (a) $\displaystyle\int_3^5 \frac{1}{x^2+x}\,dx$

   (b) $\displaystyle\int_0^2 x\sin x^2\,dx$

   (c) $\displaystyle\int_0^2 x\sin x\,dx$

5. Give the exact value of the series.

   (a) $\sum\limits_{n=0}^{\infty} \frac{1}{n!}$

   (b) $\sum\limits_{n=3}^{\infty} 3^{-n}$

6. Determine if the following series converge absolutely, conditionally or diverge. Briefly justify your answer.

   (a) $\sum\limits_{n=0}^{\infty} (-1)^n$

   (b) $\sum\limits_{n=0}^{\infty} \frac{(-1)^n}{2+n}$

7. Recall that the trapezoid rule for the integral $\int_a^b f(x)\,dx$ is

$$T_n = \frac{h}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)).$$

The error satisfies

$$\left|T_n - \int_a^b f(x)\,dx\right| \leq \frac{M_2(b-a)^3}{12n^2}$$

where M_2 is a number which is larger than the second derivative of f on the interval $[a, b]$.

(a) Compute $f''(t)$, the second derivative of $f(t) = \sin(t^2)$.

(b) Find a number M which is larger than $f''(t)$ for $0 \leq t \leq 0.4$.

(c) Find a value of n so that the error,

$$\left|T_n - \int_0^{0.4} \sin t^2\,dt\right|$$

is at most 10^{-3}.

(d) Use the trapezoid rule to estimate the integral

$$\int_0^{0.4} \sin(t^2)\,dt$$

to within 10^{-3}.

8. (a) Write down the McLaurin series for $\sin x$.

   (b) State Taylor's formula for the remainder.

   (c) Let $P_6(x)$ be the 6th degree Taylor polynomial for $\sin x$ at 0. Compute $P_6(1)$.

   (d) Use Taylor's formula for the remainder to find a number ϵ so that $|\sin(1) - P_6(1)| \leq \epsilon$.

9. (a) Eliminate the parameter, t, to find a Cartesian equation for the curve $x(t) = 3t^2 + t$ and $y(t) = 2t$.

   (b) Find the tangent line to this curve at the point $(x, y) = (14, 4)$.

10. (a) Sketch the curve $x(t) = 3\cos t$ and $y(t) = 2\sin t \cos t$ for $0 \le t \le 2\pi$.

(b) Give all values of t in $[0, 2\pi]$ for which $x(t) = y(t) = 0$.

(c) Find the equations for all tangent lines to this curve at $(0, 0)$.