Calculus II — Russell Brown

Final exam — 7 May 1999

Answer all of the following questions. Use the answer sheets provided. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may receive NO credit)*.

Name ____________________

Section __________

Question	Score	Total
1		10
2		15
3		10
4		5
5		10
6		15
7		10
8		10
9		15
Total		100

1. State the fundamental theorem of calculus, part 1 and the evaluation theorem.

2. Use calculus to evaluate the integrals.

   (a) $\displaystyle\int_{-1}^{1} \frac{1}{x^2-4}\,dx$

   (b) $\displaystyle\int_{0}^{1} xe^x\,dx$

   (c) $\displaystyle\int_{0}^{1} x\sqrt{1-x^2}\,dx$

3. Consider a sphere of radius r. (Hint: Recall that a sphere is the shape of a soccer ball.)

   (a) Show how to express the volume of the sphere as an integral by dividing the sphere into slices and taking the limit as the thickness of the slices goes to 0.

   (b) Evaluate the integral you found in part a).

4. Give the sum of the series

$$\sum_{n=2}^{\infty} 3^{-n}.$$

5. Give the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2}$$

   Be sure to check the endpoints.

6. Draw a direction field for the differential equation

$$y' = y + y^2.$$

   On your direction field, sketch a solution of the initial value problem

$$\begin{cases} y' = y + y^2 \\ y(0) = 1 \end{cases}$$

7. Use Euler's method with step-size 0.2 to estimate $y(1)$ where $y(x)$ is the solution of the initial value problem

$$\begin{cases} y' = y + y^2 \\ y(0) = 1 \end{cases}$$

8. Solve the initial value problem

$$\begin{cases} y' = y^2 \\ y(0) = 2 \end{cases}$$

9. A tank contains 400 liters of brine (salt water). Pure water flows in at a rate of 4 liter/minute and brine flows out at a rate of 4 liter/minute. Initially, the brine in the tank has a concentration of salt of 0.1 kilogram/liter.

   Assume that the tank is perfectly stirred.

   (a) Write down an initial value problem whose solution, $M(t)$, gives the mass of salt in the tank at time t.

   (b) Solve the initial value problem in part a).