

Quiz 9, 14 November 2008

1. State the factor theorem.

Solution: A number c is zero of a polynomial $P(x)$ if and only if $x - c$ is a factor $P(x)$.

2. Find the remainder if we divide

$$P(x) = x^4 + 3x^2 - 2x + 1$$

by $(x + 2)$.

Solution: Using the remainder theorem, the remainder when we divide $P(x)$ by $(x + 2) = (x - -2)$ is $P(-2) = 16 + 12 + 4 + 1 = 33$. One may also find this remainder using synthetic division.

3. Completely factor the polynomial

$$P(x) = 2x^3 + 5x^2 + 4x + 1.$$

Solution: Using the rational root theorem, the possible rational roots are $\pm 1, \pm 1/2$. Synthetic division or checking will show -1 and $-1/2$ are roots. Given one root, you have one factor. Dividing gives a quadratic polynomial. Factoring this polynomial gives the answer.

If we use synthetic division to divide by $(x + 1)$, we obtain

$$\begin{array}{r|rrrr} -1 & 2 & 5 & 4 & 1 \\ & & -2 & -3 & -1 \\ \hline & 2 & 3 & 1 & 0 \end{array}$$

Thus we have $P(x) = (x + 1)(2x^2 + 3x + 1)$ and factoring the quadratic gives $P(x) = (x + 1)(2x + 1)(x + 1)$.