Calculus I
Final exam

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Answer all of the following questions. Use the answer sheets provided. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may receive NO credit)*.

Name _____

Section _____

| Question | Score | Total |
|---|---|---|
| 1 | | 10 |
| 2 | | 10 |
| 3 | | 5 |
| 4 | | 10 |
| 5 | | 30 |
| 6 | | 10 |
| 7 | | 15 |
| 8 or 9 | | 10 |
| Total | | 100 |

1. Find the limits

   (a) $\lim_{x \to 1} \dfrac{x^2 + 2x - 3}{x - 1}$

   (b) $\lim_{x \to 0} \dfrac{\sin^2 x}{x}$

   (c) $\lim_{x \to \infty} \dfrac{3x^2 - 400x}{2x^2 + x}$

2. Compute the derivatives

   (a) $\frac{d}{dx} x^2 \sin x$

   (b) $\frac{d}{dx} \cos(x^2)$

   (c) $\frac{d}{dx} \sqrt{1 + x^2}$

3. Find the tangent line to $y = x^3 + x$ at $x = -2$.

4. A rectangle has two corners on the $x$-axis and the other two on the parabola $y = 20 - x^2$ where $y \geq 0$. What are the dimensions of the rectangle of this type with maximum area?

5. Find the following definite and indefinite integrals. The next page is blank so that you will have enough space to clearly explain your work.

   (a) $\int x\sqrt{x^2+1}\,dx$

   (b) $\int \sin x \cos x\,dx$

   (c) $\int \dfrac{x^4+1}{x^2}\,dx$

   (d) $\int_{-\pi}^{\pi} |\sin x|\,dx$

   (e) $\int_2^4 (x^2+x^3)\,dx$

   (f) $\int_0^{\pi} \cos(2x)\,dx$

6. Find the area between the curves $y = x + 2$ and $y = x^2$. As part of your solution, you should sketch the region.

7. (a) State the first fundamental theorem of calculus.
   (b) State the second fundamental theorem of calculus.
   (c) Give the proof of the second fundamental theorem of calculus.

Do one of problems 8 or 9. Indicate clearly which problem is to be graded.

8. Find the volume of a right circular cone of radius $r$ and height $h$. You may use either the method of shells or the method of disks.

9. Find the volume obtained by rotating the region between $y = x^2$ and $y = x^3$ about the $x$-axis. In your solution, you should indicate if you are using the shell or the washer method, sketch the region in the plane, give the volume of typical shell or washer, express the total volume as an integral and evaluate the integral.