Calculus I
Exam 3

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Answer all of the following questions. Use the answer sheets provided. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may receive NO credit)*.

Name _____

Section _____

| Question | Score | Total |
|---|---|---|
| 1 | | 15 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 15 |
| 5 | | 15 |
| 6 | | 10 |
| 7 | | 10 |
| 8 | | 5 |
| 9 | | 10 |
| Total | | 100 |

1. Consider the function

$$f(x) = \frac{1}{x^2 + 3}.$$

Use calculus to answer the following questions.

(a) Find the intervals where $f$ is increasing and decreasing.

(b) Find the intervals where $f$ is concave up and concave down.

(c) Find all inflection points for $f$.

(d) Sketch a graph which reflects the above information.

2. Consider $f(x) = 2x^3 + 3x^2 - 6x + 1$. Use calculus to find the (exact) values of $x$ so that $f(x)$ is a local extreme value and determine if these values of $x$ give local maximum or local minimum values. Be sure to explain how you determine if each critical point is a local maximum or local minimum.

3. Suppose $x$ and $y$ are two positive numbers whose product is 12. If possible, give maximum and minimum values for $x + y$. Be careful.

4. (a) State the mean value theorem.

   (b) Give the definition of decreasing function.

   (c) State the test which uses the first derivative to show that a function is decreasing.

   (d) Prove the first derivative test for decreasing functions.

5. Compute the anti-derivatives.

(a) $\int x^4 + 3x \, dx$

(b) $\int \frac{x^2 + 1}{x}^4 \, dx$

(c) $\int 2x(x^2 + 23)^{100} \, dx$

6. Find a function $y(x)$ which solves the differential equation and takes the specified value.

   (a) $\dfrac{dy}{dx} = x^2 + x$, $y(0) = 3$.

   (b) $\dfrac{dy}{dx} = \dfrac{x}{y}$, $y(0) = 2$.

7. Use mathematical induction to prove that

$$\sum_{k=1}^{n} 2k - 1 = n^2.$$

8. Approximate the area bounded by $y = 1/x$, $x = 1$, $x = 3$ and $y = 0$ using an inscribed polygon consisting of three rectangles with equal base.

9. This problem asks you to find the area of the region bounded by $y = x$, $x = 1$, $x = 3$ and $y = 0$. In your solution, you may need to use one or more of the following formulae.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

   (a) Write a sum which gives the area of the circumscribed polygon composed of $n$ rectangles with equal base.

   (b) Take the limit as $n$ goes to infinity of your answer to part a) to obtain the area.

   (c) Sketch the region and compute its area using elementary geometry. Use this to check your answer to part b).