MA113:025–027 Russell Brown
Final Exam 17 December 1997

Answer all of the following questions. Use the backs of the question papers for scratch paper. Additional sheets are available if necessary. No books or notes may be used. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer **(unsupported answers may receive NO credit)**, and 3) label all variables and equations.
If you use your calculator to solve an equation or produce a graph, please indicate this on your test paper. Otherwise the answer may receive no credit.

Name _____

Section _____

Question	Score	Total
1		20
2		15
3		10
4		10
5		15
6		15
7		15
Total		100

1. (a) State the definition of the derivative.

   (b) Use the definition of the derivative to find the $f'(2)$ for $f(x) = 1/x$.

2. (a) Compute the limits.

   i. $\lim_{x \to \infty} xe^{-x}$

   ii. $\lim_{x \to 0} x^2 e^x$

   iii. $\lim_{x \to 1} \dfrac{x^2 - 1}{x^2 - 2x + 1}$

   (b) Use that $\sqrt{100} = 10$ and a linear approximation to the square root function to estimate $\sqrt{98}$.

3. A red balloon and an observer are 100 feet from each other on level ground. The balloon is released and rises straight up at 8 feet/second. Find the rate of change of the balloon's angle of elevation from the observer when the balloon is 120 feet above the ground. Is the angle of elevation increasing or decreasing?

4. A function $f(x)$ is defined on $[0, 10]$. Below are graphs of the first and second derivatives of f.

   (a) Give all local maxima and minima for the function f. Explain why each point is a local minimum or local maximum.

   (b) Give the intervals where f is increasing and the intervals where f is decreasing.

   (c) Give the intervals where the graph of f is concave up and the intervals where the graph of f is concave down.

   (d) Sketch a graph of f which incorporates the information from (a)-(c).

5. A box is to be built with a square base and no top. The volume of the box will be 120 meter^3. What are the side lengths for the box with the smallest possible surface area? Be sure to explain how you know you have found the minimum.

6. (a) Below is the graph of a function f. Beginning at the value x_0 marked on the x-axis, illustrate one step of Newton's method to find x_1, the next estimate to the root of $f(x) = 0$.

(b) Show how to use Newton's method to find the negative root of the equation $x^4 - \cos x = 0$. In your answer you should give an appropriate starting point x_0, give the formula for finding x_{n+1} in terms of x_n and give approximate numerical values for $x_1,\ x_2,\ x_3 \ldots$ Your final approximate root should be correct to at least four decimal places.

7. A bicycle is travelling on a straight road. At time $t = 0$, the brakes are applied to give a constant deceleration of 6 feet/second^2. One second after applying the brakes, the bicycle's velocity is 20 feet/second.

   (a) Use antidifferentiation to find a function $h(t)$ which gives the distance travelled from the time the brakes were applied to time t.

   (b) How many seconds after applying the brakes does the bike come to a stop?

   (c) How far does the bike travel between the time the brakes are applied and the time it stops?