MA113:025–027 Russell Brown
Review 3 September 16, 1998

For the exam you must know the derivatives of sin, cos, tan, $\tan^{-1}$, $\sin^{-1}$, e^x, x^a and $\ln x$. More importantly, you should know sum, product, quotient and chain rules and the techniques of implicit differentiation and logarithmic differentiation so that you can find derivatives of interesting functions built up out of the simple functions whose derivatives you have memorized.

1. Differentiate the following functions:
   (a) $(x+1)e^x$
   (b) $(x^2+3x)e^{\sin x}$
   (c) $(x^3+\sin x)^2$
   (d) $\cos(e^x)$
   (e) $\tan^{-1}(1+x^2)$
   (f) $e^x \cos x$
   (g) $\frac{x^2+1}{\sin x}$
   (h) $x^{\sin x}$

2. A ball is thrown in the air. At a time t seconds after it is thrown, the ball's height is $h(t) = 30t - 4.9t^2$ meters.
   (a) What are the units for $h'(t)$?
   (b) When is the ball moving up?
   (c) When does the ball return to the ground?
   (d) What is the total distance that the ball travels? (Hint: The answer is not 0.)

3. Use the quotient rule and the derivatives of sin and cos to find

$$\frac{d}{dx}\sec x, \ \frac{d}{dx}\csc x, \ \frac{d}{dx}\tan x, \ \frac{d}{dx}\cot x.$$

4. If $f'(x) = g(x)$, $g(x) < 0$ and $g(x)^2 - f(x)^2 = 1$, find

$$\frac{d}{dx} f^{-1}(x).$$

   Hint: Imitate the procedure for finding $\frac{d}{dx}\sin^{-1}(x)$.

5. Use implicit differentiation to find

$$\frac{d}{dx}\cot^{-1} x.$$

   Here $\cot^{-1}$ is the inverse of function $\cot x$ for x in the interval $(0, \pi)$.

6. Suppose we define $f(x)$ to be the inverse function to $\sin x$ for x satisfying $\pi/2 \leq x \leq 3\pi/2$.

   (a) Sketch the graph of $f(x)$. Give the domain and range of f.

   (b) Find the derivative of $f(x)$. (Hint: This function is not the function $\sin^{-1} x$ which is discussed in the text. The derivative might be different.)

7. For which x does the tangent line to $y = x^2 + 1$ pass through the origin?

8. Find all the tangent lines to the parametric curve

$$x(t) = \sin(2t), \; y(t) = \cos(t)$$

   which pass through $(x, y) = (0, 0)$. (Hint: Graph the curve to find out how many tangent lines the curve has.)

9. Find all tangent lines at $(x, y) = (0, 0)$ for the parametric curve

$$x(t) = t^2 + t, \; y(t) = t^3 - t.$$

10. Find the tangent line(s) to the curve $x^2 + y = y^3$ when $y = 2$.

11. A function $y = f(x)$ satisfies the equation $x^2 + y^2 = 2xy$. Find the linear approximation to f at $(x, y) = (1, 1)$. Use this linear approximation to estimate $f(0.9)$.

12. (a) Find where the curves $x^2 + y^2 + 1$ and $x^2 - 2x + 1 + y^2 = 1$ intersect. (Exact answers please.)

    (b) Do the curves intersect orthogonally?

13. Give the linear approximation of $f(x) = \sin(\sin(x))$ at $x = 0$. Use your answer to estimate $f(0.1)$.

14. Use a linear approximation to estimate $\sqrt{101}$.

15. Evaluate the limits:

    (a) $\lim_{x \to 0} \frac{1 - \cos x}{\sin(x^2)}$

    (b) $\lim_{x \to \infty} e^{-x} x^2$

    (c) $\lim_{x \to -\infty} e^{-x} x^2$

    (d) $\lim_{x \to 0^+} x^x$

    (e) $\lim_{x \to 2} \frac{x - 2}{x + 2}$

    (f) $\lim_{x \to 0^+} (1 + 2/x)^x$.