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Review 4 September 16, 1998

The final will be cumulative with an emphasis on Chapter 4. Below are review problems for Chapter 4. You will also want to study the review problems for the first three exams.

1. Consider the function $f(x) = x^4 - x^2$. Find all critical numbers. Determine if each critical number is a local maximum, local minimum or neither. Find local maximum values and local minimum values. Find intervals of increase or decrease. Find intervals of concavity. Find inflection points. Sketch a graph using this information. Check using TI-82.
2. Suppose a light is revolving at two revolutions/min. The light is three meters from a wall. a) Find the speed with which the light is moving along the wall when the beam forms a right angle with wall. b) Find the speed with which the light is moving along the wall if the light forms an angle of 45° with the wall.
3. A rectangle is inscribed in the ellipse $2x^2 + y^2 = 1$. Find the largest possible area of the rectangle.
4. Find the global maximum and minimum values of $f(x) = \sin x - \frac{1}{2}x$ on $[0, \pi]$.
5. Suppose that a prism whose base is an equilateral triangle has volume 100 cm^3. Find the small possible surface area.

6. If we apply Newton's method to the following function starting a D, which root will Newton's method find? Same question for E and F.

7. Explain Newton's method and derive the formula

$$x_{n+1} = x_n - f(x_n)/f'(x_n).$$

8. Show how use Newton's method to find $\sqrt[3]{10}$. Give x_1, x_2 and x_3 if $x_0 = 4$.

9. State the first derivative test for a global minimum. Use this test to show that $f(x) = x + 1/x$ has a global minimum on $(0, \infty)$.

10. State the mean value theorem. State the definition of increasing function. Use the mean value theorem to prove that if $f'(x) > 0$ for $x \in (0, 1)$, then f is increasing on $(0, 1)$.

11. Let $f(x) = x^3 - x$ for x in $[-2, 2]$. Find the global maximum and minimum. What are the global maximum and minimum values.