

MA113:025-027  
Exam 1

Russell Brown  
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Answer all of the following questions. Use the backs of the question papers for scratch paper. Additional sheets are available if necessary. No books or notes may be used. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer (**unsupported answers may receive NO credit**), and 3) label all variables and equations. If you use your calculator to solve an equation or produce a graph, please indicate this on your test paper. Otherwise the answer will be assumed to be “unsupported”.

Name \_\_\_\_\_

Section \_\_\_\_\_

1. (14 points)

- (a) If  $f(x) = x^2$  and  $g(x) = 2x + 1$ . Give  $f \circ g$  and  $g \circ f$ .
- (b) Find an exact value for  $\log_2 8$ . Do not use your calculator.
- (c) If  $h(x) = x^3 - x$ , is  $h$  even, odd or neither?

2. (14 points) A rectangular window has two dimensions  $l$  and  $w$ .
- (a) Assume that the perimeter of the window is 30 feet. Give the area as a function of the  $l$ . Give the domain of this function.
  - (b) Suppose the area of the window is 45 square feet. Give an approximate value for the dimensions of the window. Your answer should be correct to the first decimal place.

3. (14 points) Consider the graph below.

- (a) Find  $f(1)$ ,  $f(2)$  and  $f \circ f(4)$ .
- (b) Does  $f^{-1}$  exist? If so, find  $f^{-1}(1)$ . If not, explain why not.
- (c) On the axes provided, sketch  $2f(x)$ . Label the coordinates of the point on the graph where  $2f(x)$  is largest.
- (d) On the axes provided, sketch the graph of  $f(-x + 1)$ . Label the coordinates of the point on the graph where  $f(-x + 1)$  is largest.

4. (14 points) Suppose that a particle moves along the path given by the parametric equations:

$$\begin{aligned}x(t) &= 2t + 1 \\y(t) &= t^2 - 4t.\end{aligned}$$

- (a) Eliminate the parameter to find an equation in  $x$  and  $y$  which each point on the curve satisfies. Simplify your answer.
- (b) Find the value of  $t$  where the  $y$  coordinate is smaller than at any other point on the curve.
- (c) Find the  $x$  and  $y$  coordinates of the point where the  $y$  coordinate is smaller than at any other point on the curve.

5. (14 points) Suppose

$$f(x) = 2 + \frac{1}{x-1}.$$

- (a) Give a sketch of the graph of  $f$ . (You may use your calculator.)  
Why does  $f^{-1}$  exist?
- (b) Give a formula for  $f^{-1}(x)$ .
- (c) Give the domain of  $f$  and the range of  $f$ .

6. (15 points) Suppose a population grows exponentially. At  $t = 0$ , the population has 100 critters. After 2 days, the population has doubled.
- (a) Give formula for  $N(t)$ , the size of the population after  $t$  days.
  - (b) Find the population after 5 days.
  - (c) Find when the population reaches 1000 critters.

7. (15 points) Consider the table of numbers.

$x$	$y$
1	2
2	2.83
4	4.00
8	5.66
12	6.92

- (a) Make a plot of  $\ln x$  against  $\ln y$  on the axes.
- (b) Estimate the slope and  $y$ -intercept of the line which passes near the points you plotted in part a).
- (c) Using your answers in part b), find an equation of the form  $y = ax^r$  which the numbers  $x$  and  $y$  satisfy. You must explain how you found your answer. You may not use the power regression function on your calculator. (I suppose you could use this function to check your answer, but I expect that there are easier ways to check your answer.)