

Answer all of the following questions. Use the backs of the question papers for scratch paper. Additional sheets are available if necessary. No books or notes may be used. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer (**unsupported answers may receive NO credit**), and 3) label all variables and equations. If you use your calculator to solve an equation or produce a graph, please indicate this on your test paper. Otherwise the answer may receive no credit.

Name _____

Section _____

| Question | Score | Total |
|----------|-------|-------|
| 1 | | 20 |
| 2 | | 15 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 15 |
| 6 | | 15 |
| 7 | | 15 |
| Total | | 100 |

1. For each of the following functions, find the derivative. Do not simplify your answer.

(a) $x^2 + \ln x$

(b) $\frac{1 - t^2}{1 + t^2}$

(c) $x \cos(1/x)$

(d) $(x^2 + 1)e^{2x}$

(e) $x^{\cos x}$

2. A mass is attached to a spring and slides without friction along the x -axis. Its position is given by the function $x(t) = 2 \cos(t)$ where $x(t)$ denotes its displacement in centimeters from the origin at time t seconds.

- (a) For which t in $[0, 2\pi]$ is $x(t) \geq 0$.
- (b) What are the units of $x'(t)$?
- (c) For which t in $[0, 2\pi]$ is the mass moving to the right?
- (d) What is the total distance travelled for $0 \leq t \leq 2\pi$? (Hint: The answer is not zero.)

3. Use the quotient rule and the derivatives of sin and cos to find

$$\frac{d}{dx} \cot x.$$

Simplify your answer.

4. Let $f(x) = \tan x$ for $\pi/2 < x < 3\pi/2$.

(a) Sketch the graph of $f(x)$ and $f^{-1}(x)$.

(b) Find

$$\frac{d}{dx} f^{-1}(x).$$

Simplify in order to express your answer as a rational function of x .

5. Consider the parametric curve

$$x(t) = t - \sin(\pi t/2), \quad y(t) = t^2 - t.$$

- (a) Find the two tangent lines at $(x, y) = (0, 0)$.
- (b) Do these tangent lines cross at a right angle?

6. Let $y(x)$ satisfy the equation $x^2 + y^3 = xy$.

(a) Find the linear approximation to $y(x)$ at $(x, y) = (2, -2)$.

(b) Use your answer to part a) to estimate the values of $y(x)$ when $x = 2.2$ and $x = 1.95$.

7. Find the limits, if they exist. Indicate how you obtain each limit. Answers obtained by guessing based on calculated values will not receive full credit.

(a) $\lim_{x \rightarrow 0} \frac{x^3}{\sin(x)}$

(b) $\lim_{x \rightarrow 0} \frac{\cos x}{x}$

(c) $\lim_{x \rightarrow 0^+} (1 + x)^{3/x}$