

Below is a selection of problems related to sections 3.3 and 3.4. These problems will not be collected or graded. However, you should understand how to work each of these problems. You should begin working on these problems in groups in recitation. You will probably want to finish these problems outside of class. If you have questions, please ask your TA or instructor. If you find a problem difficult, consider working similar problems from the text for additional practice.

Announcements: 1. There is homework due Wednesday in class. 2. The next written homework assignment is §3.3 #4, §3.4 #26, 38, §3.5 #22, 38, 48. It will be due on Wednesday, 4 November. 3. Next spring, Brown will be teaching MA114, section 003–005 at 10 am MWF. See the schedule book for recitation times. TA's have not been assigned for the spring quarter.

1. Here is an exercise regarding tangents to a parabola. Several of you have asked how many tangent lines to a parabola pass through a given point. As with most questions, the answer is “it depends”. In this exercise, you are to see what “it depends” means in a particular case.
 - (a) Consider the parabola which is the graph of the function $f(x) = x^2$. Let $(1, b)$ denote a point on the vertical line where the x coordinate is 1. For the moment, let b be fixed, and consider the tangent line to the graph of $f(x)$ at $x = a$. Write out a quadratic equation in a whose solutions are the values of a for which the tangent line at $x = a$ passes through the point $(1, b)$. Of course, b will appear in this equation.
 - (b) Now imagine that b is varying and ask for which values of b , the quadratic equation in a has 0, 1, or 2 solutions. (Hint: Use the quadratic formula. The number of (real) solutions of a quadratic equations depends on the sign of the expression inside the square root sign. This expression is usually called the discriminant.)
 - (c) Your instructor thinks that if $b = 1$, there is one tangent line through $(1, b)$. If $b < 1$, there are two tangent lines through $(1, b)$ and if $b > 1$, there are no tangent lines to the parabola which pass through $(1, b)$. Is he right? (Alert students will note that this is different from the answer I gave in class.) Draw a sketch to help you visualize the different cases.
 - (d) Consider the same problem with the point $(1, b)$ replaced by an arbitrary point in the (x, y) plane, (m, n) . What condition on (m, n) guarantee that there are two tangent lines to the parabola which pass through (m, n) ? Does your answer make sense?
2. Work the following problems from §3.3. #1,3,5,7,12,13. In studying this section, concentrate on the examples from geometry, physics and chemistry.

3. In your answers to 5 and 7 in section 3.3, can you give a geometric interpretation for the rate of change of area? What are the units for this derivative?
4. Work the following problems from §3.4. #1, 3, 9, 11, 13, 15, 21, 25, 27, 29, 33, 34, 37, 38, 41.
5. In section 3.4, you must memorize the derivatives of sin and cos. Using these derivatives, the definitions of sec, tan, csc and cot and the product, reciprocal and quotient rule, you should be able to figure out the remaining derivatives.
6. In this section, you must memorize the basic fact that

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1.$$

Using this you should be able to find related limits such as

$$\lim_{x \rightarrow 0} \frac{\sin(-2x)}{x} \quad \lim_{x \rightarrow 0} \frac{\tan 2x}{x} \quad \lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta}$$

Note that you can use your calculator to draw a graph and check your answer. Your instructor reserves the right to ask you to justify your answer by using limit laws and the definitions of trig functions in terms of sin and cos.

7. Below is a multi-part problem to see why

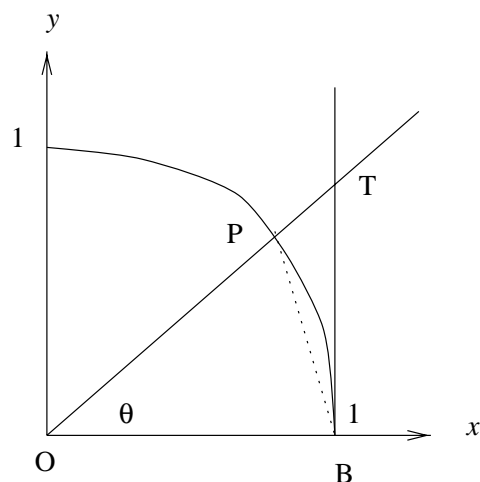
$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1.$$

This exercise will not appear on the examination. However, some of you may find it interesting and it will provide a review of trigonometry and the squeeze theorem which will be useful in this course.

The argument presented below is different from the one in the text.

- (a) Is the function $\sin \theta / \theta$ even or odd or neither? Explain why it is sufficient to only consider the one-sided limit:

$$\lim_{\theta \rightarrow 0^+} \frac{\sin \theta}{\theta}.$$



- (b) Since we are interested in the limit as $\theta \rightarrow 0^+$, only values of θ near 0 are relevant. Thus, we may assume that $0 < \theta < \pi/2$ and thus the point $P = (\cos \theta, \sin \theta)$ lies in the first quadrant, as pictured.
- (c) Let S be the area of the sector of the unit circle determined by the the points O , B and P (Hint: If $\theta = 2\pi$, then the area of the sector is the area of the unit circle. Use proportionality, to find the area of S .) Let K_1 be the area of the triangle BOP and let K_2 be the area of the triangle BOT . You should express these areas in terms of the θ and the trigonometric functions $\sin \theta$ and $\cos \theta$. Order the numbers S , K_1 and K_2 from smallest to largest using the $<$ sign.
- (d) Use a little bit of algebra and two inequalities from your answer to the previous part to find two functions f and g which satisfy

$$f(\theta) < \frac{\sin \theta}{\theta} < g(\theta), \quad \text{if } 0 < \theta < \pi/2$$

Apply the squeeze theorem.

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