MA113:004–006 Review 1

The first hour exam is from 7:30 to 9:30 pm in CB 114 on Tuesday, 22 September. You may use a calculator during the exam. You may not use a laptop computer a calculator which can perform symbolic manipulation, such as the TI-92. If you have any questions as to whether your calculator is allowed, please see Brown before the test.

The exam will cover sections 1.1 to 1.6 of Stewart and the principle of mathematical induction. There will be no questions taken from appendix C. (Though the trigonometric functions do arise in problems from Chapter 1.)

To review for the exam, you should review all assigned homework, quizzes, the problem sheets from recitation and material from the book and lecture. The review sheet below provides additional problems for you to work.

- 1. Let  $f(x) = \sqrt{x-4}$  and  $g(x) = x^2 + 1$ . Give the domain of f. Find  $f \circ g$  and  $g \circ f$ .
- 2. Sketch the graph of f(x) = 1/x. Based on this graph, find graphs of f(x+1), -2f(x) and f(-x+1) 1.
- 3. Suppose that a wheel of radius 1 meter rolls along the x-axis without slipping. If the bottom of the wheel is initially at the origin, find the coordinates of the point P after the wheel has rotated through an angle of  $\pi/4$ . Hint: How far has the wheel travelled after it has rotated through an angle of pi/4.
- 4. Consider the parametric curve

 $x = \sin t, \qquad y = \cos t.$ 

- (a) Make a graph of the curve and give an arrow to indicate the direction we move as t increases.
- (b) Given an equation in x and y that every point on the curve satisfies.
- (c) Find a value of t where the y coordinate is as large as possible.
- 5. A population doubles every 10 days and after 20 days, the population is 400.
  - (a) What is the population initially?
  - (b) Give the size of the population after 23 days.
  - (c) When does the population reach 20,000?
- 6. (a) Solve  $2^{x^2} = 7$ .
  - (b) Find the exact value of  $\log_3 1/9$ .
- 7. Sketch the graph of this function  $f(x) = \sin x$  with domain  $[\pi/2, 3\pi/2]$ . Give the range of this function. Graph  $f^{-1}$  and give its domain and range. Find  $f^{-1}(0)$ . Why is your answer different than your calculator's value for  $\sin^{-1}(0)$ ?
- 8. State the principle of mathematical induction.

- 9. Use induction to prove that  $5^n 1$  is a multiple of 4 for n = 1, 2, ...
- 10. Use induction to prove that  $\sum_{k=1}^{n} 4k = 2n^2 + 2n$  for  $n = 1, 2, \dots$
- 11. If f is odd, is  $f \circ f$  odd? Is 2f odd? Is  $f^2$  odd?
- 12. Which of the following functions are even?

$$f(x) = x^4 + x^2$$
,  $g(x) = x^5 - x^2$ ,  $h(x) = \cos x \sin x$ .