MA113:004–006 Russell Brown

Review 2 October 14, 1998

The second hour exam is on Tuesday, 20 October from 7:30pm–9:30pm in CB 114. You may use a graphing calculator on the exam. You may not use notes, textbooks, a computer or a calculator with symbolic manipulation capabilities such as a TI-92.

Below are a selection of problems to help you prepare for the exam. You should also review material covered in lecture and the problems assigned for recitation.

1. Estimate the following limits. Your answers should be correct to 2 decimal places.

$$\lim_{x\to 0} \sin(e^x - 1)/x, \qquad \lim_{x\to 0} \frac{2^x - 3^x}{x}.$$

2. Find the following limits using the limit laws, if they exist. Explain each step of your solution. If a limit does not exist, determine if it ∞ or $-\infty$.

$$\lim_{x\to 2} \frac{x-2}{x^2-4}, \qquad \lim_{x\to 2} \frac{x+2}{x^2+4}, \qquad \lim_{x\to 2} \frac{x+3}{x-2}.$$

3. Find the following limits, if they exist. If the limit does not exist, explain why.

$$\lim_{x\to 0} \frac{|x|}{x}, \qquad \lim_{x\to\infty} \frac{x^2+3}{3x^2+4x}.$$

4. Use the squeeze theorem to evaluate the following limits. $\lim_{x\to\infty} \frac{\sin x}{x}$, $\lim_{x\to 0} x\sin(1/x)$.

5. Let $f(x) = 2^x$. Using the definition, estimate the derivative at $x = 2$.

6. Let $f(x) = \sqrt{2x}$. Find the derivative, $f'(x)$, using the definition. Give the domain of the derivative.

7. State the definition of the derivative of f at a.

8. If f is differentiable at a, is f continuous at a? If your answer is yes, provide a proof. If your answer is no, provide an example.

9. If f is continuous at a, is f differentiable at a? If your answer is yes, provide a proof. If your answer is no, provide an example.

10. On which intervals are the following functions continuous? On which intervals are the functions differentiable.

$$f(x) = |x|, \qquad g(x) = \frac{1}{x^2-4}.$$

11. Show that the equation $\cos x = x$ has a solution.

12. Let $f(x) = e^{2x}$. a) Estimate $f'(0)$. Explain how you arrived at your answer. b) Use the tangent line at 0 to approximate $e^{1.02}$ and $e^{0.99}$.

13. If the prevailing price for milk is p dollars per gallon, then a store will sell $Q(p)$ gallons of milk. a) What are the units for $Q'(p)$? b) Do you expect $Q'(p) > 0$ or $Q'(p) < 0$? Why?

14. State the intermediate value theorem.

15. Draw the graph of a function which is defined, but not continuous, at $x = 2$ and which is continuous, but not differentiable, at $x = 4$.

16. Below are the graphs of f, f' and f''. Which is which? Explain your answer.

17. Below is the graph of a function f. Sketch the graph of f'.

18. Below is the graph of a function f'. Sketch two possibilities for the graph of f.