MA113:004–006 Russell Brown
Review 4 December 2, 1998

The final will be primarily over the sections we have covered in Chapter 4, 4.1, 4.2, 4.3, 4.5, 4.6, 4.8 and 4.9. (Note related rates, 4.1 is repeated from the third test.) However, you must know how to differentiate, that is you must know the product rule, quotient rule, chain rule and the derivatives of the elementary functions. In addition, I may give one question asking you to use implicit differentiation to find a derivative of an inverse trigonometric function, a logarithm or of $\sinh^{-1}$. For this reason, I have repeated the review questions on this topic which appeared on the review sheet for exam 3.
Reminder: There is a study session from 10–12 am in the Math House on Monday, 14 December 1998. The final is from 8:30–10:30pm on Monday, 14 December 1998 in CB122.

1. Consider the function $f(x) = x^4 - x^2$. Find all critical numbers. Determine if each critical number is a local maximum, local minimum or neither. Find local maximum values and local minimum values. Find intervals of increase or decrease. Find intervals of concavity. Find inflection points. Sketch a graph using this information. Check using TI-82.

2. Suppose a light is revolving at two revolutions/min. The light is three meters from a wall. a) Find the speed with which the light is moving along the wall when the beam forms a right angle with wall. b) Find the speed with which the light is moving along the wall if the light forms an angle of 45° with the wall.

3. Suppose an airplane is directly east of you and flying to the east at an altitude of 4 miles and a rate of 300 miles per hour. Give the rate of change of the angle of elevation with respect to time when the plane is 150 miles to the east. You may assume the earth is flat.

4. A rectangle with sides parallel to the axes is inscribed in the ellipse $2x^2 + y^2 = 1$. Find the largest possible area of the rectangle.

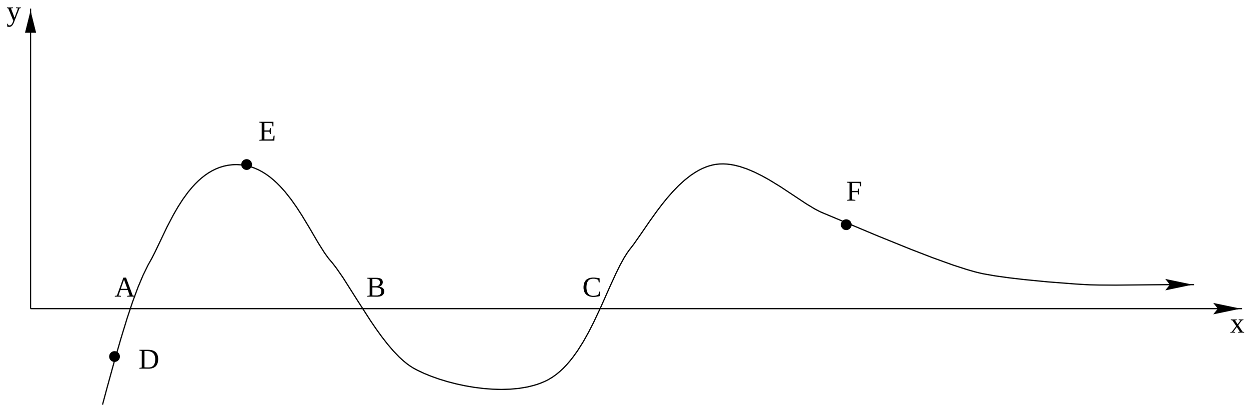
5. Find the absolute maximum and minimum values of $f(x) = \sin x - \frac{1}{2}x$ on $[0, \pi]$.

6. Suppose that a prism whose base is an equilateral triangle has volume 100 cm^3. Find the small possible surface area.

7. Find the limits:

$$\lim_{x\to\infty}\frac{e^x}{x} \qquad \lim_{x\to 0^+} x^{2x} \qquad \lim x \to 0\frac{\sin x}{x+1}.$$

8. If we apply Newton's method to the following function starting a D, which root will Newton's method find? Same question for E and F.



9. Explain Newton's method and derive the formula

$$x_{n+1} = x_n - f(x_n)/f'(x_n).$$

10. Show how use Newton's method to find $\sqrt[3]{10}$. Give x_1, x_2 and x_3 if $x_0 = 4$.

11. State the first derivative test for a absolute minimum. Use this test to show that $f(x) = x + 1/x$ has a absolute minimum on $(0, \infty)$.

12. Let $f(x) = x^3 - x$ for x in $[-2, 2]$. Find the absolute maximum and minimum. What are the absolute maximum and minimum values.

13. If $f'(x) = g(x)$, $g(x) < 0$ and $g(x)^2 - f(x)^2 = 1$, find

$$\frac{d}{dx}f^{-1}(x).$$

Hint: Imitate the procedure for finding $\frac{d}{dx}\sin^{-1}(x)$.

14. Use implicit differentiation to find

$$\frac{d}{dx}\cot^{-1} x.$$

Here $\cot^{-1}$ is the inverse of function $\cot x$ for x in the interval $(0, \pi)$.

15. Suppose we define $f(x)$ to be the inverse function to $\sin x$ for x satisfying $\pi/2 \le x \le 3\pi/2$.

   (a) Sketch the graph of $f(x)$. Give the domain and range of f.

   (b) Find the derivative of $f(x)$. (Hint: This function is not the function $\sin^{-1} x$ which is discussed in the text. The derivative might be different.)