MA113:004–006 Russell Brown
Exam 3 17 November 1998

Answer all of the following questions. Use the backs of the question papers for scratch paper. Additional sheets are available if necessary. No books or notes may be used. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may receive NO credit)*.
If you use your calculator to solve an equation or produce a graph, please indicate this on your test paper. Otherwise the answer may receive no credit.

Name ________________________

Section ____________

Question	Score	Total
1		25
2		10
3		15
4		10
5		10
6		15
7		10
8		15
Total		100

1. Differentiate the following functions. You do not need to simplify your answer.

   (a) $e^x + \sin x$

   (b) $\frac{1-x}{1+x}$

   (c) $(x^2+1)^5$

   (d) $\sin(5x)\cos x$

   (e) $\sin^{-1}(2x)$

2. Find the tangent line to the curve

$$x^2 + 5x^2y^2 + y^3 = 13$$

at the point $(x, y) = (1, -2)$.

3. Use induction to prove that

$$\frac{d}{dx} x^n = n x^{n-1}, \qquad n = 1, 2, \ldots$$

You may assume that the formula is true for $n = 1$.

4. Find the derivative of the function

$$f(x) = x^{2x}.$$

5. Use the derivatives of $\cos x$ and $\sin x$ to find the derivative of

$$\cot x.$$

Simplify your answer.

6. Use the value of the limit, $\lim_{t\to 0} \sin t/t$, (which you should know) and the limit rules to find

   (a) $\displaystyle \lim_{x\to 0} \frac{\sin(5x)}{x}$

   (b) $\displaystyle \lim_{y\to 0} \frac{\tan\theta}{\theta}$

   (c) $\displaystyle \lim_{z\to 0} \frac{1-\cos z}{z^2}$

7. Find the tangent line to the parametric curve

$$x(t) = \cos t, \qquad y(t) = 2\sin t$$

at $t = \pi/4$.

8. On Friday 15 October 1998, Logan and Ian are standing at a point X in the northwest part of Lexington. Logan is wearing a red sweater and Ian is wearing a blue sweater. At 12 noon, Logan begins running north at 5 miles per hour and Ian begins running east at 4 miles per hour. Let θ be the angle given by X, Ian's position and Logan's position (see sketch). What is the rate of change of θ with respect to time at 12:30 pm, Friday, 15 October 1998.