

The goal of this assignment is to consider the hyperbolic trigonometric functions. Many of the properties of these functions are similar to those of the more familiar (circular) trigonometric functions. Thus, it would be boring to cover these functions in lecture. A more interesting¹ approach is to allow you to work through these properties on your own. This has two benefits: 1) You will learn a little bit about the hyperbolic trigonometric functions, so you will not be surprised if you run into them in a future course. 2) Many of the techniques used in studying these functions are identical to techniques used in studying the ordinary trigonometric functions. Thus, you will understand the trigonometric functions better after you complete this assignment. Alert students will note that many of the things discussed in this assignment are in section 7.8. Please do not just copy from this section. I expect that many of you will find the explanations in section 7.8 rather brief and will want to write out a more detailed explanation in order to fully understand what you are doing.

Parts of this assignment are taken from the textbook, *Calculus: Concepts and contexts* by J. Stewart.

As with all longer assignments, you may work in a group of 1 to 3 students. You should begin this assignment in recitation on Thursday, 10 February and hand in your carefully written solutions on Friday, 18 February. This assignment will be worth 20 points.

The hyperbolic sine and cosine are denoted by \sinh and \cosh and are defined by

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad \cosh x = \frac{e^x + e^{-x}}{2}.$$

Then, in analogy with the ordinary trigonometric functions, we define hyperbolic tangent and hyperbolic secant by:

$$\tanh x = \frac{\sinh x}{\cosh x} \quad \operatorname{sech} x = \frac{1}{\cosh x}.$$

The functions \coth and csch could also be defined, but we will not use them in this assignment.

1. Sketch the graphs of $\sinh x$, $\cosh x$ and $\tanh x$, possibly using a graphing calculator. What are the domains and ranges of these functions?
2. The most famous application of the hyperbolic cosine arises in describing the shape of a hanging wire such as a power line. This shape is called a catenary and is given by the graph of $a \cosh(x/a)$. Graph this curve for several different $a > 0$ and describe how the curve changes.
3. Simplify $\cosh^2 x - \sinh^2 x$. You should obtain a constant and thus prove the hyperbolic analogue of the Pythagorean identity for \sin and \cos .

¹One of my axioms of teaching is that whenever the professor says something is interesting, the students are in trouble. I hope this assignment contradicts this axiom.

4. Graph the parametric curve given by $x(t) = \cosh t$ and $y(t) = \sinh t$. What curve is this? (Hint: It is not a parabola.)
5. Simplify $2 \sinh x \cosh x$. Which trigonometric identity does the resulting identity resemble?
6. Find the derivatives

$$\frac{d}{dx} \sinh x, \quad \frac{d}{dx} \cosh x, \quad \frac{d}{dx} \tanh x.$$

In each case, you should be able to express the derivative in terms of the hyperbolic trigonometric functions. The results are similar to the ordinary trigonometric functions.

7. Find an identity relating sech^2 and \tanh^2 . Hint: Divide the identity you proved in part 3 by \cosh^2 .
8. (a) Explain why \tanh is one to one and thus that the inverse function, \tanh^{-1} , exists.
 - (b) Use implicit differentiation to find the the derivative of $y = \tanh^{-1} x$. You may want to model your answer on the procedure we used to find the \tan^{-1} in section 7.7.
 - (c) Show that $\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$.
 - (d) Compute the derivative of \tanh^{-1} using the result of part c). Compare your answer with part b). Do you get the same answer?

There is a way to define the (ordinary) trigonometric functions in terms of the exponential functions. If i is the square root of -1 , then

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \quad \cos x = \frac{e^{ix} + e^{-ix}}{2}.$$

With these definitions, it is even clearer that there should be many similarities between \sin and \cos and \sinh and \cosh . Of course, the disadvantage is that we have to know about complex numbers and exponentials of complex numbers. Thus, we will not pursue this approach.²

²But it would be very interesting!