

MA 114, Section 21
Project 2 (25 points)
Due August 3, 2000

Name: _____

You may have heard about the Fibonacci sequence of numbers. It starts with the numbers

$$1, 1, 2, 3, 5, 8, 13, \dots$$

this sequence was introduced around A.D. 1200 by Leonardo Fibonacci. We can define this sequence $\{f_n\}$ by the recursive formula

$$f_1 = f_2 = 1, \quad f_{n+2} = f_{n+1} + f_n.$$

However, this definition is not very useful if we need to know what f_{223} is. In this project, we will find an explicit formula for f_n . Complete the steps below.

1. Find f_1 through f_{12} .
2. Consider the power series $F(x) = f_1x + f_2x^2 + f_3x^3 + \dots$. Show that the radius of convergence of this power series is at least $\frac{1}{2}$.
3. Find a second degree polynomial $p(x)$ such that $p(x)F(x) = x$.
4. Write a partial fraction decomposition for $\frac{x}{p(x)}$.
5. Write the power series expansion for $\frac{1}{ax+b}$ and determine its radius of convergence.
6. Write the power series expansion for your partial fraction decomposition for $\frac{x}{p(x)}$ and use it to determine the coefficients for $F(x)$.
7. What is the radius of convergence for $F(x)$?
8. Use your result in problem 6 to determine a formula for f_k for each value of k . Use this formula and a calculator to find the first 8 terms of the sequence $\{f_n\}$ and verify your answers by comparing them with problem 1.
9. Find f_{223} .

You will be expected to justify each statement you make. For example, when finding $p(x)$ above, you must convince me that $p(x)F(x) = x$. Don't just tell me what $p(x)$ is. The same rules apply as last time:

You should work in groups of 1 to 3 students. Each group will hand in one report for the group and all group members will receive the SAME GRADE. By signing your name to the report you submit, you are attesting that you participated in the work and that you accept the grade of the group. When you use results from the text (or from other sources) be sure to give precise reference by page and equation number.

You should feel free to ask for help in my office hours or by e-mail. Little to no time will be spent on this in class. You are free to consult with others outside your group; however, you should not copy directly from another paper, obviously.