MA162

The following topics and skills will be examined in the second midterm test.

- Graphing the feasible set of a system of linear inequalities in two variables. Finding corner points
- Formulating written descriptions of feasible sets as linear inequalities.
- Solving linear programs by graphical, corner point and simplex methods.
- Formulate word problems as linear programs.
- The augmented coordinate system.
- The simplex tableau: Basic and nonbasic variables, choosing the pivot element, finding the augmented coordinates of a corner point and determining if a tableau corresponds to the optimal feasible point.
- Dual prices, righthand side ranging and changing coefficients in objective function.

Below are some sample questions to help you prepare for the exam. Be sure that you understand all of the topics from the above list, not just the topics covered by the questions below! In your answers on the exam, please be sure to 1) Check answers when possible. 2) Clearly indicate your answer and the reasoning used to arrive at that answer. 3) Label all variables and equations.

1. (a) Solve the following linear program by the graphical method.

$$\begin{array}{ll} \max & x+2y\\ \mathrm{st} & x+y \leq 4\\ & 2x+y \leq 6\\ & x \geq 0, \ y \geq 0 \end{array}$$

- (b) Check your answer by re-solving the problem by the corner point method.
- 2. The feasible set for the following system of linear inequalities is sketched below.

$$egin{array}{rcl} x-y &\leq & 0 \ 5x+6y &\leq & 30 \ y &\leq & 4 \ x \geq 0, & y \geq 0 \end{array}$$



- (a) Rewrite the system of inequalities by introducing slack variables.
- (b) Find the augmented coordinates of the points labelled A-E.
- You own a shipping company which transports two types of packages, A and B. Your plane can carry only 100 units of volume and 60 units of weight.

You know that a package of type A has a volume of 5 units and a weight of 1 unit, but will give a profit of \$3 a package. A package of type B has a volume of 2 units and a weight of 2 units and gives you a profit of \$2 per package. Customs will allow you to transport no more than 16 packages of type A per flight.

How many of each type package should you transport to maximize your profit? What is the maximum profit?

4. (a) Carry out one simplex pivot on the following tableau.

P	x_1	x_2	s_1	s_2	RHS
1	-1	-2	0	0	0
0	1	1	1	0	4
0	2	4	0	1	8

After you have done the pivot, answer the following questions:

i. Are we done with the problem, that is have we reached an optimal solution? Why or why not?

- ii. Find the associated corner point
- (b) Carry out one pivot so that the variable x_2 becomes basic. (Remark: There are two possible ways to do this.)

x_1	x_2	x_3	s_1	s_2	<i>s</i> 3	RHS
1	-1	0	0	1	-2	4
0	-3	0	1	1	0	3
0	0	1	0	-2	1	2

After you have pivoted, give the basic variables and their values. What are the nonbasic variables and their values?

- 5. Use the attached LINDO printout to answer the following questions.
 - (a) What is the optimal feasible point? What is the maximum value of the objective function?
 - (b) Compute the new value of the objective function if the coefficient of x is decreased by 1 unit.
 - (c) Compute the new value of the objective function if the coefficient of x in the objective function is 8.
 - (d) Compute the new value of the objective function if the righthand side of constraint 2 is increased by 10 units. (For this question, assume that the coefficient of x has been returned to its original value, 4.)

```
4 X + 2 Y + 5 Z
MAX
SUBJECT TO
       2)
            X + 2 Y + 3 Z <=
                                 30
       3)
            3 X + Y + 2 Z <=
                                 40
       4)
            X + Y + Z <=
                             20
END
LP OPTIMUM FOUND AT STEP
                                2
       OBJECTIVE FUNCTION VALUE
       1)
              70.0000000
 VARIABLE
                  VALUE
                                  REDUCED COST
        Х
                   8.571428
                                       0.000000
        Y
                   0.000000
                                       1.000000
        Ζ
                   7.142857
                                       0.000000
```

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.00000	1.000000
3)	0.00000	1.000000
4)	4.285714	0.00000

NO. ITERATIONS= 2

3 4

RANGES IN WHICH THE BASIS IS UNCHANGED:

40.000000

20.000000

		OBJ COEFFICIENT RANGES	3
VARIABLE	CURRENT	ALLOWABLE	ALLOWABLE
	COEF	INCREASE	DECREASE
Х	4.000000	3.500000	2.333333
Y	2.000000	1.000000	INFINITY
Z	5.000000	6.999999	1.400000
		RIGHTHAND SIDE RANGES	
ROW	CURRENT	ALLOWABLE	ALLOWABLE
	RHS	INCREASE	DECREASE
2	30.00000	30.000000	16.666664

15.000000

INFINITY

20.000000

4.285714