The following topics and skills will be examined in the third midterm test.

- Trees, the multiplication principle, Venn diagrams and the inclusion-exclusion principle and counting problems.
- Permutations, combinations and distinguishable permutations. You will need to know the formulas and be able to recognize which formula is needed in a particular problem.
- Computation of probability using counting techniques. Use of the complement rule and the formula for the probability of the union of two events.
- Definitions of conditional probability and independent events.
- Bayes formula. The use of a tree in organizing the information for Bayes theorem.

Below are some sample questions to help you prepare for the exam. Be sure that you understand all of the topics from the above list, not just the topics covered by the questions below! In your answers on the exam, please be sure to 1) Check answers when possible. 2) Clearly indicate your answer and the reasoning used to arrive at that answer. 3) Label all variables and equations.

1. Is it always true that $B \cap (A \cup C)' = (A' \cup B) \cap (A' \cup C)$? Draw two Venn diagrams to justify your answer.
2. Five thousand people answered a survey about vacations spots. Of these, 1655 liked Florida, 1155 liked California and 955 liked Hawaii. Both California and Florida were preferred by 105, 55 liked both California and Hawaii and 80 liked both Hawaii and Florida. There were 30 people who liked all three places.
   (a) How many liked just Hawaii?
   (b) How many liked Florida and California, but not Hawaii?
   (c) How many did not like any of the three?
3. (a) If a lottery has 40 numbers and you choose six of these for each play, how many possible tickets are there? Assume that the order does not matter.
   (b) How many distinguishable permutations are there of the letters in 'COMMITTEE'?
   (c) Given that the number of combinations of 5 distinct objects taken two at a time is 5!/(3!2!), explain why the number of ways of arranging the 5 letters in 'ALOOF' is 5!/2.

(d) Seventy people belong to the math club. How many different ways can we choose a speaker committee of 4 people and a publicity committee of 6. Assume that no one is expected to serve on more than one committee.

4. A coin is flipped. If the coin is heads, you choose a number between 1 and 3. If the coin is tails, you choose a number between 1 and 4. Draw a tree to summarize this probability experiment and answer the following questions.

   (a) List the elements in the sample space for this game. Are all the outcomes equally likely?

   (b) What is the probability of obtaining a 1?

   (c) What is the probability of obtaining a 4?

   (d) What is the probability of obtaining an even number?

5. If a coin is flipped 5 times, compute the probability of

   (a) obtaining no heads.

   (b) obtaining at least one head.

   (c) obtaining exactly one head.

6. A survey of 100 students gives the following information.

   - 55 took calculus
   - 40 took finite mathematics
   - 15 did not take calculus and did not take finite mathematics

   (a) Find the number of students who took calculus and finite mathematics.

   (b) Given that a student took calculus, find the probability that the student also took finite mathematics.

   (c) Find the probability that a student took finite mathematics or calculus.

   (d) According to the above information, are the events of taking finite mathematics and calculus independent?

7. Suppose that you draw a number from 1,2,3, 4, 5. If you draw an odd number, you receive that amount in dollars. If you draw an even number, you must pay that amount. What is the expected value of playing this game?

8. Suppose that 30% of patients with a sore throat have a bacterial infection. Suppose that a throat culture indicates bacterial infection in 10% of patients without bacterial infections and does not indicate bacterial infection in 15% of those who have a bacterial infection.

(a) Draw a tree to summarize the above information.

(b) What is the probability that a patient with a sore throat will have a positive culture?

(c) Suppose that a patient has a positive culture. What is the probability that the patient has a bacterial infection?

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