Solutions for some problems from A2, A3.

>

> **with(linalg):**

Homework A2

Q.2.

The demand equation for the OK 1020 All in One office machine is x+5p=500 where x is the quantity demanded per week and p is the wholesale unit price in dollars. The supply equation is x-40p+3000=0 where x is the number of machines the supplier will make available each week when the wholesale price is p dollars per machine. Find the equilibrium quantity and equilibrium price for these machines.
Equilibrium quantity x = machines, and equilibrium price = dollars.
Round your answer for x down to the nearest integer.
First we solve the demand and supply equations together. Then answer the questions.

The solution for x is $\frac{1000}{9}$ **rounded down to 111 and the price is reported as 700/9 - no rounding required!**

> **solve({x+5*p=500,x-40*p+3000=0},{x,p});**

$$\{p = \frac{700}{9}, x = \frac{1000}{9}\}$$

> **floor(1000/9);**

$$111$$

Q.5.

A manufacturer can produce 7,500 units for a total profit of 26,800.00 dollars, but if he increases his production to 10,500 units, then his profit becomes $38,800.00.

It follows that his fixed cost is $

and his net profit per unit produced is $
Note: We are assuming that the profit is a linear function of the production.
We assume the profit function is P(x)= a+bx. Then we are told that 26800=a+7500*b and 38800=a+10500*b.

Subtracting first from second, we get: 12000=3000*b, so b=4 and then from the first equation, we get 26800=a+7500*4 or 26800=30000. Thus a=-3200.

The fixed cost is then $3200 or "-a" and profit per unit is "b" or $4.

```
> solve({26800=a+7500*b,38800=a+10500*b},{a,b});
```

$\{a = -3200, b = 4\}$

Q.6.

i) The two lines -4x-y=-18, and 3x+4y=8 intersect at the point where x=
and y=
ii) If the line 3x+ky=8 is parallel to -4x-y=-18, then what is the value of k?   Answer: k =

For the first part, we just solve the two equations together and report the answers.

$\{x = \frac{64}{13}, y = -\frac{22}{13}\}$

.

For the second part, we have a neat method.
From Cramer's rule to solve equations ax+by=c, px+qy=r, we know that the only time the equations fail to have a solution is when the determinant $\Delta = \begin{bmatrix} a & b \\ p & q \end{bmatrix}$ is zero.

Our calculation gives $\Delta = -3 + 4k$ and this is zero exactly when $k = \frac{3}{4}$.

```
> solve({-4*x-y=-18,3*x+4*y=8},{x,y});
```

$\{x = \frac{64}{13}, y = \frac{-22}{13}\}$

```
> matrix(2,2,[3,k,-4,-1]);
```

$$\begin{bmatrix} 3 & k \\ -4 & -1 \end{bmatrix}$$

```
> det(%);
```

$-3 + 4k$

> Delta = matrix(2,2,[a, b, p, q]);

$$\Delta = \begin{bmatrix} a & b \\ p & q \end{bmatrix}$$

Q.9.

A truck purchased for $67,000.00 is to be linearly depreciated to $8,700.00 in 8 years using the straight-line method. What would be the book value of the truck after 5 years ?

ANSWER: Your answer should be correct to two decimal places.

The function is different from the one in lectures where the value was depreciated to zero. So you need to make your formula!

You assume the value at time t to be linear v(t) = a+bt. The given information says that v(0) = 67000=a+b(0)=a.
Also, v(8)=8700=a+b(8).

Thus we know that a=67000 and from a+8b=8700, we get $b = \frac{8700 - 67000}{8}$ **or** $-\frac{14575}{2}$**. Then the desired answer is** $a + b\,5 = 67000 - \frac{14575\,(5)}{2}$ **which evaluates to** $\frac{61125}{2}$**.**
Note that we don't have to actually give a decimal answer, the precise fraction is even better!

> (8700-67000)/8;

$$\frac{-14575}{2}$$

> 67000-14575/2*5;

$$\frac{61125}{2}$$

>

>

Homework A3

Q.1.
The only value of b for which the following system does not have a unique solution is: b =

3v+2z=0   -t-2v-2z=-1   -2t+bv+3z=-2
Moreover, for this value of b the system of equations is inconsistent/has infinitely many solutions.

We set up the augmented matrix and start row reductions. We have kept the variables order as t,v,z however, other orders are possible.

```
> A:=matrix(3,4,[0,3,2,0,-1,-2,-2,-1,-2,b,3,-2]);
```

$$A := \begin{bmatrix} 0 & 3 & 2 & 0 \\ -1 & -2 & -2 & -1 \\ -2 & b & 3 & -2 \end{bmatrix}$$

p.p. is (2,1,1), so we swap rows 1 and 2.

```
> A1:=swaprow(A,1,2);
```

$$A1 := \begin{bmatrix} -1 & -2 & -2 & -1 \\ 0 & 3 & 2 & 0 \\ -2 & b & 3 & -2 \end{bmatrix}$$

Now we use pivot -1 in row 1 to kill target -2 in row 3.

```
> A2:=addrow(A1,1,3,-2);
```

$$A2 := \begin{bmatrix} -1 & -2 & -2 & -1 \\ 0 & 3 & 2 & 0 \\ 0 & 4+b & 7 & 0 \end{bmatrix}$$

Now the p.p. is (1,2,2) and we use the pivot 3 in row to to kill the target 4+b in row 3.

```
> A3:=addrow(A2,2,3,-(4+b)/3);
```

$$A3 := \begin{bmatrix} -1 & -2 & -2 & -1 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & \frac{13}{3} - \frac{2b}{3} & 0 \end{bmatrix}$$

Time to inspect. The p.p. is now (1,2,3) if $\frac{13}{3} - \frac{2b}{3}$ is non zero and ($1, 2, \infty$) if it is zero.
If the p.p. is (1,2,3), then we have a unique solution since no free variable is left!
So, we want the value of b which makes the quaantity $\frac{13}{3} - \frac{2b}{3}$ equal to zero!
This gives b=13/2.

Moreover, for this value of b, we see that we have only two pivots and hence a free variable. So, we have infinitely many solutions!

Q.2. A value of the number k for which there is either no solution or an infinite number of solutions (s ,x ,y) to the system of equations shown below is:
-4s-x-2y=-2 3s-2x+ky=-4 -4s-4x+y=1

Again, we make the augmented matrix and start row reductions.
Note that we have chosen the variable order x,s,y for easier operations!

```
> A:=matrix(3,4,[-1,-4,-2,-2,-2,3,k,-4,-4,-4,1,1]);
>
```

$$A := \begin{bmatrix} -1 & -4 & -2 & -2 \\ -2 & 3 & k & -4 \\ -4 & -4 & 1 & 1 \end{bmatrix}$$

p.p. is (1,1,1) so, we make two cleanup operations.

```
> A1:=addrow(A,1,2,-2);
```

$$A1 := \begin{bmatrix} -1 & -4 & -2 & -2 \\ 0 & 11 & 4+k & 0 \\ -4 & -4 & 1 & 1 \end{bmatrix}$$

> **A2:=addrow(A1,1,3,-4);**

$$A2 := \begin{bmatrix} -1 & -4 & -2 & -2 \\ 0 & 11 & 4+k & 0 \\ 0 & 12 & 9 & 9 \end{bmatrix}$$

Now p.p. is (1,2,2), so we make one more cleanup.

> **A3:=addrow(A2,2,3,-12/11);**

$$A3 := \begin{bmatrix} -1 & -4 & -2 & -2 \\ 0 & 11 & 4+k & 0 \\ 0 & 0 & \frac{51}{11} - \frac{12\,k}{11} & 9 \end{bmatrix}$$

If the entry $\frac{51}{11} - \frac{12\,k}{11}$ **is non zero, then p.p. would be (1,2,3) and we have a unique solution. So, we solve for this entry to be zero. This gives** $k = \frac{51}{12}$**. Now for this value of k, the last row becomes (0 0 0 9) which makes an inconsistent equation.**

Q.3.

Given the initial augmented matrix $\begin{bmatrix} 1 & -1 & 0 & 3 \\ 3 & -2 & -2 & 4 \\ -2 & 2 & 1 & -4 \end{bmatrix}$ **find it RREF.**

Does the corresponding system of equations have no solution, a unique solution or more than 1 solution?
We write the matrix and work on it. We are given a partial answer, but we use it only to compare.

> **A:=matrix(3,4,[1, -1, 0, 3, 3, -2, -2, 4, -2, 2, 1, -4]);**

$$A := \begin{bmatrix} 1 & -1 & 0 & 3 \\ 3 & -2 & -2 & 4 \\ -2 & 2 & 1 & -4 \end{bmatrix}$$

> **A1:=addrow(A,1,2,-3);**

$$A1 := \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -2 & -5 \\ -2 & 2 & 1 & -4 \end{bmatrix}$$

> **A2:=addrow(A1,1,3,2);**

$$A2 := \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

We have REF. Now we start cleaning above the pivots (which are luckily all 1).

> **A3:=addrow(A2,3,2,2);**

$$A3 := \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

> **A4:=addrow(A3,2,1,1);**

$$A4 := \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Since this is an augmented matrix, there are three varaibles. Since the p.p. is (1,2,3), there are no free variables.
Hence a unique solution!

Q.4. The reduced row echelon form of the matrix

$$\begin{bmatrix} -1 & 2 & -\frac{3}{2} \\ -1 & -4 & \frac{1}{2} \\ -1 & -1 & -\frac{1}{2} \end{bmatrix}$$

is:

We just start reductions.

```
> A:=matrix(3,3,[-1,2,-3/2, -1,-4,1/2,-1,-1,-1/2 ]);
```

$$A := \begin{bmatrix} -1 & 2 & \frac{-3}{2} \\ -1 & -4 & \frac{1}{2} \\ -1 & -1 & \frac{-1}{2} \end{bmatrix}$$

```
> A1:=addrow(A,1,2,-1);
```

$$A1 := \begin{bmatrix} -1 & 2 & \frac{-3}{2} \\ 0 & -6 & 2 \\ -1 & -1 & \frac{-1}{2} \end{bmatrix}$$

```
> A2:=addrow(A1,1,3,-1);
```

$$A2 := \begin{bmatrix} -1 & 2 & \frac{-3}{2} \\ 0 & -6 & 2 \\ 0 & -3 & 1 \end{bmatrix}$$

> **A3:=addrow(A2,2,3,-1/2);**

$$A3 := \begin{bmatrix} -1 & 2 & \frac{-3}{2} \\ 0 & -6 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

We have REF with p.p. ($1, 2, \infty$). The pivots are 1,-6. We turn them into 1 and clean above them.

> **A4:=mulrow(A3,2,-1/6);**

$$A4 := \begin{bmatrix} -1 & 2 & \frac{-3}{2} \\ 0 & 1 & \frac{-1}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

> **A5:=mulrow(A4,1,-1);**

$$A5 := \begin{bmatrix} 1 & -2 & \frac{3}{2} \\ 0 & 1 & \frac{-1}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

> **A6:=addrow(A5,2,1,2);**

$$A6 := \begin{bmatrix} 1 & 0 & \frac{5}{6} \\ 0 & 1 & \frac{-1}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

This is RREF. Note that there are only two pivots and they are both 1 with unit columns.

Q.5.
In the traffic flow diagram below, find the solution with x1, x2, x3, x4 all non-negative and x4 as small as possible.
We are asked to solve for all variables.
At each node of the diagram, we write the condition that inflow minus outflow equals zero.
The equations are:
670+1320-x1-x4=0
x1+x2-620-1170=0

x3+x4-1300-700 =0

and

400+1400 - x2-x3=0

We make an augmented matrix and solve.

```
> A:=matrix(4,5,[1,0,0,1,1990,1,1,0,0,1790, 0,0,1,1,2000,0,1,1,0,1800]);
```

$$A := \begin{bmatrix} 1 & 0 & 0 & 1 & 1990 \\ 1 & 1 & 0 & 0 & 1790 \\ 0 & 0 & 1 & 1 & 2000 \\ 0 & 1 & 1 & 0 & 1800 \end{bmatrix}$$

```
> A1:=addrow(A,1,2,-1);
```

$$A1 := \begin{bmatrix} 1 & 0 & 0 & 1 & 1990 \\ 0 & 1 & 0 & -1 & -200 \\ 0 & 0 & 1 & 1 & 2000 \\ 0 & 1 & 1 & 0 & 1800 \end{bmatrix}$$

```
> A2:=addrow(A1,2,4,-1);
```

$$A2 := \begin{bmatrix} 1 & 0 & 0 & 1 & 1990 \\ 0 & 1 & 0 & -1 & -200 \\ 0 & 0 & 1 & 1 & 2000 \\ 0 & 0 & 1 & 1 & 2000 \end{bmatrix}$$

```
> A3:=addrow(A2,3,4,-1);
```

$$A3 := \begin{bmatrix} 1 & 0 & 0 & 1 & 1990 \\ 0 & 1 & 0 & -1 & -200 \\ 0 & 0 & 1 & 1 & 2000 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that we have achieved and REF and in fact, by luck an RREF!

The p.p. is ($1, 2, 3, \infty$ **) So, we can write down the solution with x1,x2,x3 as pivot variables and x4 as free.**

Answers are:
x1=1990-x4, x2=x4-200, x3=2000-x4.

The condition that all variables are non negative says x4 is at least 200 and at most 1990.

Since we want it to be as small as possible we take x4=200 and get the answer:
x1=1790, x2=0, x3=1800, x4=200.

Q.6.

We want to invest a total of $45,000 in two funds A and B that have yields of 6% and 8% interest per year, respectively. We want the total interest received after 1 year to be $3,300.00. How much should be invested in each fund?
Amount invested in Fund A : **Amount invested in Fund B :**

Let us name the investment in fund A as x and in fund B as y.
This gives two equations:
x+y=45000 **Total investment compared.**

0.06x+0.08y = 3300 **Total yield compared.**
You may rewrite the second equation as 6x+8y=330000 by multiplying by 100, if desired.
Solution gives x=15000, y=30000.

```
> solve({x+y=45000,0.06*x+0.08*y=3300},{x,y});
```

$$\{y = 30000., x = 15000.\}$$

Q.7.

A value of the number k for which there is either no solution or an infinite number of solutions (x ,z) to the system of equations shown below is:
4x+kz=4,   6x-7z=-7

We attempt to use Cramer's rule as above and note that the determinant Δ becomes

$$4(-7) - 6k = -28 - 6k$$

.

In order to get infinitely many solutions or no solutions, we must have Δ equal to zero.

$$k = -\frac{28}{6} \text{ or } -\frac{14}{3}$$

This gives the answer .

Alternatively, we can proceed to solve the equation using row reductions as shown below.

```
> A:=matrix(2,3,[4,k,4,6,-7,7]);
```

$$A := \begin{bmatrix} 4 & k & 4 \\ 6 & -7 & 7 \end{bmatrix}$$

```
> A1:=addrow(A,1,2,-6/4);
```

$$A1 := \begin{bmatrix} 4 & k & 4 \\ 0 & -\frac{3k}{2} - 7 & 1 \end{bmatrix}$$

Now, if the quantity $-\frac{3k}{2} - 7$ is nonzero, then we have two pivots and would get a uniques solution! So, it must be zero. Setting it to zero and solving gives $k = -\frac{14}{3}$.

Q.8.
If b = then the following system of equations is consistent.

3y+r=13,   y-2r=b,   -2y+r=-4

We set up an augmented matrix and start solving. The consistency condition will give an equation satisfied by b which we solve.

> **A:=matrix(3,3,[3,1,1,3,-2,b,-2,1,-4]);**

$$A := \begin{bmatrix} 3 & 1 & 1 \\ 3 & -2 & b \\ -2 & 1 & -4 \end{bmatrix}$$

> **A1:=addrow(A,1,2,-1);**

$$A1 := \begin{bmatrix} 3 & 1 & 1 \\ 0 & -3 & -1+b \\ -2 & 1 & -4 \end{bmatrix}$$

> **A2:=addrow(A1,1,3,2/3);**

$$A2 := \begin{bmatrix} 3 & 1 & 1 \\ 0 & -3 & -1+b \\ 0 & \frac{5}{3} & \frac{-10}{3} \end{bmatrix}$$

> **A3:=addrow(A2,2,3,5/9);**

$$A3 := \begin{bmatrix} 3 & 1 & 1 \\ 0 & -3 & -1+b \\ 0 & 0 & -\frac{35}{9}+\frac{5b}{9} \end{bmatrix}$$

Now the consistency condition says $-\frac{35}{9}+\frac{5b}{9}$ **must be zero, so** $b = 7$.

Q.9.

A portfolio manager will invest \$1000,000 in three funds: x dollars in a low-risk fund with a return of 7% per year, y dollars in a medium-risk fund with a return of 8% per year, and z

dollars in a high risk fund with a return of 11% per year. He has decided to distribute the entire amount between the funds so that his total return will be $90,000.00 = 9% of 1000,000.

The two natural equations in x, y, and z that you can write down with the given information are:

i) x + y + z = 1000000

and ii) 0.07 x + 0.08 y + 0.11 z = 90000

You would expect this system of equations to have infinitely many solutions

We have filled in the answers above (in red) by guessing that the first equation compares the total investment and the second one compares the return.

Since these are only two equations in three variables we conclude that we expect at least one free variable and hence infinitely many solutions, if there are any at all!

Q.10.

The system

3s-x+3y=3, -3s-4x-3y=-3, -s+7x-y=-1

has no solution, more than one solution, unique solution.

If we use the hint that the REF is $\begin{bmatrix} 3 & -1 & 3 & 3 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **then it is clear that the third variable is free and the equations are consistent. So, we have more than one solutions (infinitely many, in fact).**
If we don't use the hint, then we have to get our own REF as shown below.

```
> matrix(3,4,[3,-1,3,3,0,05,0,0,0,0,0,0]);
```

$$\begin{bmatrix} 3 & -1 & 3 & 3 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> A:=matrix(3,4,[3,-1,3,3,-3,-4,-3,-3,-1,7,-1,-1]);
```

$$A := \begin{bmatrix} 3 & -1 & 3 & 3 \\ -3 & -4 & -3 & -3 \\ -1 & 7 & -1 & -1 \end{bmatrix}$$

> **A1:=addrow(A,1,2,1);**

$$A1 := \begin{bmatrix} 3 & -1 & 3 & 3 \\ 0 & -5 & 0 & 0 \\ -1 & 7 & -1 & -1 \end{bmatrix}$$

> **A2:=addrow(A1,1,3,1/3);**

$$A2 := \begin{bmatrix} 3 & -1 & 3 & 3 \\ 0 & -5 & 0 & 0 \\ 0 & \frac{20}{3} & 0 & 0 \end{bmatrix}$$

> **A3:=addrow(A2,2,3,4/3);**

$$A3 := \begin{bmatrix} 3 & -1 & 3 & 3 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus, we got the same REF! Incidently, REF is not unique. It is OK to get a different answer as long as the number of pivots and the consistency condition is the same.

Q.11.

A building contractor is planning to build an apartment complex with one, two or three bedroom apartments. Let x,y,z respectively denote the number of apartments of each type to be built. Suppose that the builder will spend a total of $4,644,000, and that the costs for the three types of apartments are $17,000, $28,000, and $44,000 respectively.
Then the 'cost equation' for the builder is:

17 x + 28 y + 44 z = 4644
Suppose he plans to build a total of 168 apartments, then complete the equation in x, y, and z which describes this:

(1) x + (1) y + z = 168
Further suppose that the number of one bedroom apartments must equal the total number of bigger apartments. Then we get the restriction:

x +(-1)y +(-1)z = 0.

Finally, find the solution to these equations: x= 84 , y = 30 , and z = 54

We have filled in the answers in red using the given information and sensing the nature of the equations.
The final solution has to be found by solving the nequations on your own.

```
> solve({17*x+28*y+44*z=4644, x+y+z=168, x-y-z=0},{x,y,z});
```

$$\{z = 54, y = 30, x = 84\}$$

You could do this with an augmentd matrix as shown below. Note that we have changed the order of equations for convenience of operations.

```
> A:=matrix(3,4,[1,-1,-1,0, 1,1,1,168, 17,28,44,4644]);
```

$$A := \begin{bmatrix} 1 & -1 & -1 & 0 \\ 1 & 1 & 1 & 168 \\ 17 & 28 & 44 & 4644 \end{bmatrix}$$

```
> A1:=addrow(A,1,2,-1);
```

$$A1 := \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 2 & 2 & 168 \\ 17 & 28 & 44 & 4644 \end{bmatrix}$$

```
> A2:=addrow(A1,1,3,-17);
```

$$A2 := \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 2 & 2 & 168 \\ 0 & 45 & 61 & 4644 \end{bmatrix}$$

```
> A3:=addrow(A2,2,3,-45/2);
```

$$A3 := \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 2 & 2 & 168 \\ 0 & 0 & 16 & 864 \end{bmatrix}$$

We proceed to RREF for a complete solution, instead of using the back substitution.

> **A4:=mulrow(A3,3,1/16);**

$$A4 := \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 2 & 2 & 168 \\ 0 & 0 & 1 & 54 \end{bmatrix}$$

> **A5:=addrow(A4,3,2,-2);**

$$A5 := \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 2 & 0 & 60 \\ 0 & 0 & 1 & 54 \end{bmatrix}$$

> **A6:=addrow(A5,3,1,1);**

$$A6 := \begin{bmatrix} 1 & -1 & 0 & 54 \\ 0 & 2 & 0 & 60 \\ 0 & 0 & 1 & 54 \end{bmatrix}$$

> **A7:=mulrow(A6,2,1/2);**

$$A7 := \begin{bmatrix} 1 & -1 & 0 & 54 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 54 \end{bmatrix}$$

> **A8:=addrow(A7,2,1,1);**

$$A8 := \begin{bmatrix} 1 & 0 & 0 & 84 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 54 \end{bmatrix}$$

The solution is now visible in the last column !

Q.12.

Given the initial augmented matrix- $\begin{bmatrix} -1 & 1 & 3 & -1 \\ 2 & -1 & -4 & 2 \\ 2 & -2 & -6 & 2 \end{bmatrix}$ **find the RREF**

Does the corresponding system of equations have no solution, a unique solution or more than 1 solution?

The answer is computed below. We choose more than one solutions since a free varaible is present.

```
> A:=matrix(3,4,[-1,1,3,-1,2,-1,-4,2,2,-2,-6,2]);
```

$$A := \begin{bmatrix} -1 & 1 & 3 & -1 \\ 2 & -1 & -4 & 2 \\ 2 & -2 & -6 & 2 \end{bmatrix}$$

```
> A1:=addrow(A,1,2,2);
```

$$A1 := \begin{bmatrix} -1 & 1 & 3 & -1 \\ 0 & 1 & 2 & 0 \\ 2 & -2 & -6 & 2 \end{bmatrix}$$

```
> A2:=addrow(A1,1,3,2);
```

$$A2 := \begin{bmatrix} -1 & 1 & 3 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

> **A3:=addrow(A2,2,1,-1);**

$$A3 := \begin{bmatrix} -1 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

> **A4:=mulrow(A3,1,-1);**

$$A4 := \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

>

>

>

>

>