Exam # 1

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. (15 points) Find the center and radius of the circle with equation:

$$x^2 + 6x + y^2 - 4 = 0.$$

(*Hint:* Complete the square with x.)

$x^2 + 6x + y^2 - 4 = 0$

$(x^2 + 2 \cdot 3x + 9) - 9 + y^2 - 4 = 0$

$(x+3)^2 + y^2 - 9 - 4 = 0$

$(x+3)^2 + y^2 = 13$ (i.e. $(x+3)^2 + (y+0)^2 = (\sqrt{13})^2$)

So the center of the circle is $\boxed{(-3, 0)}$

and the radius is $\boxed{\sqrt{13}}$.

2. (5 points) What is the 1^{st} operation applied to x in the following expression

$$13 - (x-4)^2.$$

(a) Subtract it from 13
(b) Multiply by -1
(c) Raise it to the 2^{nd} power
(d) Take the square root
(e) Subtract 4

Note: remember operations in parentheses are prioritised.

3. (15 points) Solve for x in the equation

$$\sqrt{16x+41} = x+5.$$

Square both sides first:

$$\left(\sqrt{16x+41}\right)^2 = (x+5)^2$$

$$16x+41 = x^2+10x+25$$

$$0 = x^2-6x-16$$

Using quadratic formula we have:

$$x = \frac{6 \pm \sqrt{(-6)^2-4\cdot 1\cdot(-16)}}{2\cdot 1} = \frac{6\pm\sqrt{36+64}}{2} = \frac{6\pm\sqrt{100}}{2} = \frac{6\pm 10}{2}$$

so $x_1 = \frac{6+10}{2} = \frac{16}{2} = 8$ and $x_2 = \frac{6-10}{2} = \frac{-4}{2} = -2$.

Check: $\sqrt{16\cdot 8+41} \stackrel{?}{=} 8+5$ and $\sqrt{16\cdot(-2)+41} \stackrel{?}{=} -2+5$

$\sqrt{169} \stackrel{?}{=} 13$ $\sqrt{9} = 3$

$13 = 13$ ✓ $3 = 3$ ✓

Solutions: $\boxed{x=8}$ and $\boxed{x=-2}$

4. (10 points) Find the x-intercepts and y-intercepts of the following equation:

$$y^2 - 4y - 21 = x.$$

x-intercept: $y=0 \rightarrow 0^2-4\cdot 0-21 = x$

$-21 = x$

so, x-intercept at $\boxed{(-21,0)}$.

y-intercept: $x=0 \rightarrow y^2-4y-21=0$

$(y-7)(y+3)=0$

so $y-7=0$ or $y=7$ and $y+3=0$ or $y=-3$.

so, y-intercept at $\boxed{(0,7)}$ and $\boxed{(0,-3)}$

5. (15 points) Solve for x:

$$x^6 - 14x^3 = -49.$$

Let $u = x^3$, then $u^2 = x^6$ and

$$u^2 - 14u + 49 = 0.$$

$$(u-7)^2 = 0$$

$$u - 7 = 0$$

$$u = 7$$

Thus, $x^3 = 7$ and $\boxed{x = \sqrt[3]{7}}$.

6. (7 points) Give an equation of the line through $(4, 1)$ that is parallel to $4x + 2y - 2 = 0$.

Step 1

Find the slope of the line $4x + 2y - 2 = 0$

$$2y = -4x + 2$$

$$y = -2x + 1 \quad \text{(so } -2 \text{ is the slope)}.$$

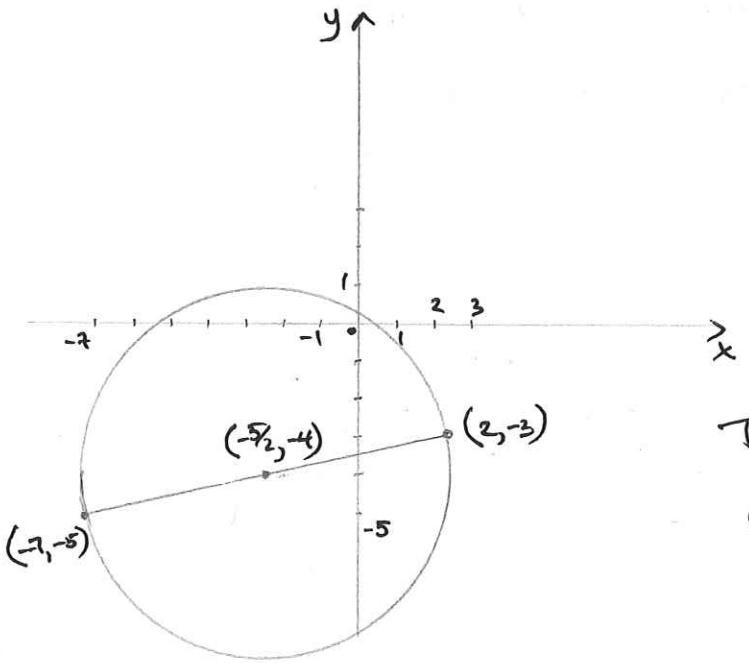
Since we are talking about parallel line then we are going to have the same slope.

So new line has slope -2 and passes through the point $(4, 1)$, thus equation of the line is

$\boxed{y - 1 = -2(x-4)}$ or $\boxed{y = -2(x-4) + 1}$

or $\boxed{y = -2x + 9}$

7. (8 points) What is the center of the circle which has a diameter with endpoints at $(-7, -5)$ and $(2, -3)$?



Diameter of the circle is a line that passes through the center of the circle and midpoint of this line is exactly the center of the circle.

Thus

Center is $\left(\frac{-7+2}{2}, \frac{-5-3}{2}\right) = \boxed{\left(-\frac{5}{2}, -4\right)}$

8. (10 points) Find a solution to the following system of equations.

$$3x - 2y = 2$$
$$5x + y = 4$$

Take the second equation and multiply it by 2.

$$\begin{array}{r} 3x - 2y = 2 \\ + \quad 10x + 2y = 8 \\ \hline 13x + 0 = 10 \end{array}$$

Using method of elimination. So $13x = 10$ or $x = \frac{10}{13}$.

Now find y.

$$5 \cdot \frac{10}{13} + y = 4 \longrightarrow \frac{50}{13} - 4 = -y$$

$$\frac{50 - 4 \cdot 13}{13} = -y$$

$$-\frac{2}{13} = -y \quad \text{so} \quad y = \frac{2}{13}.$$

Solution: $\boxed{\left(\frac{10}{13}, \frac{2}{13}\right)}$.

9. (15 points) Solve the system:

$$(x+3)^2 + y^2 = 8$$
$$x + y = -3$$

Solve 2nd equation for y, $y = -3 - x$, thus

$$(x+3)^2 + (-3-x)^2 = 8$$
$$(x+3)^2 + (-(x+3))^2 = 8$$
$$(x+3)^2 + (x+3)^2 = 8$$
$$2(x+3)^2 = 8$$
$$(x+3)^2 = 4$$

$x+3 = 2$ or $x+3 = -2$

$x = 2-3$ $\quad$ $x = -2-3$

$= -1$ $\quad$ $= -5$

when:

$x = -1 \rightarrow -1 + y = -3$

$y = -3 + 1 = -2$

$x = -5 \rightarrow -5 + y = -3$

$y = -3 + 5 = 2$

Solution:

$(-1, -2)$ and $(-5, 2)$.

10. (10 points) BONUS: A student has exam scores of 92, 70, 52, and 78. What score does he need on the fifth exam to have an average of 78?

Let x = score that our student needs on the 5th exam to have an average of 78.

Then

$$\frac{92+70+52+78+x}{5} = 78$$

$$\frac{292+x}{5} = 78$$

$$292 + x = 390 \quad \rightsquigarrow \quad x = 98.$$

Conclusion:

Student needs to get a score of 98 on the 5th exam to have an average of 78.

Name: Key

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	15	5	15	10	15	7	8	10	15	10	110
Score:	15	5	15	10	15	7	8	10	15	10	110