Exam # 2

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. (10 points) Interpret the following in terms of distance on the number line:

$$|x+6| > 4.$$

1. x is more than 4 units away from 6.
2. x is more than 4 units away from -6.
3. x is more than 6 units away from 4.
4. x is more than 6 units away from -4.
5. No solutions.

2. (15 points) Solve the following inequality for x. Express your answer in interval notation.

$$x^2 - 10x \leq -21$$

1) $x^2 - 10x \leq -21 \longrightarrow x^2 - 10x + 21 \leq 0$.

2) Consider equality $x^2 - 10x + 21 = 0$

3) Finding solution to equality:

$$x^2 - 10x + 21 = 0 \longrightarrow (x-7)(x-3) = 0$$

4) Number line:

5) Since $x^2 - 10x + 21 \leq 0$, we need interval with $-$ sign.

So solution is $\boxed{[3,7]}$.

3. (8 points) Decide which of the following tables could describe a function. If a table could describe a function decide if it is one-to-one.

Input	-1	3	17	-1	11
Output	4	π	4	π	6

  - Is not a function since input -1 yields output 4 and π. Since not a function, thus not 1:1.

Input	-5	14	7.2	5	7
Output	9	3	9	6	1

  - Is a function, different inputs, but not 1:1 since -5 input gives 9 and 7.2 input gives 9 as well. So horizontal line test is not passed.

Input	-3	4	-9	14	7
Output	1	6	1	-7	-14

  - Is a function, different inputs, but not 1:1 since -3 gives 1 and -9 gives 1 thus horizontal line test is not passed.

4. (10 points) The function f is defined piecewise in the following way:

$$f(x) = \begin{cases} -x+1 & \text{if } x \leq -3 \\ x^2+2 & \text{if } -3 < x \leq 4 \\ 9 & \text{if } x > 4. \end{cases}$$

Find the following:

- $f(-10) =$ $-(-10)+1 = 10+1 = \boxed{11}$ since $-10 \leq -3$
- $f(-3) =$ $-(-3)+1 = 3+1 = \boxed{4}$ since $-3 \leq -3$
- $f(0) =$ $0^2+2 = \boxed{2}$ since $-3 < 0 \leq 4$
- $f(4) =$ $4^2+2 = 16+2 = \boxed{18}$ since $-3 < 4 \leq 4$
- $f(300) =$ $\boxed{9}$ since $300 > 4$.

5. (10 points) Let $f(x) = x^2 - 3x + 1$. Calculate the difference quotient $\dfrac{f(x+h) - f(x)}{h}$. **Simplify.**

1) $f(x+h) = (x+h)^2 - 3(x+h) + 1 = x^2 + 2xh + h^2 - 3x - 3h + 1$

2) $$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - 3x - 3h + 1 - (x^2 - 3x + 1)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 1 - x^2 + 3x - 1}{h}$$

$$= \frac{2xh + h^2 - 3h}{h}$$

$$= \frac{h(2x + h - 3)}{h}$$

$$= \boxed{2x + h - 3}$$

6. (10 points) Find the average rate of change of $f(x) = x^2 - 3x + 1$ as x changes from -2 to 3.

1) $f(-2) = (-2)^2 - 3(-2) + 1 = 4 + 6 + 1 = 11$

$f(3) = 3^2 - 3 \cdot 3 + 1 = 9 - 9 + 1 = 1$

2) Aver. rate of change $= \dfrac{f(3) - f(-2)}{3 - (-2)}$

$$= \frac{1 - 11}{5}$$

$$= \frac{-10}{5} = \boxed{-2}$$

7. (10 points) Find the inverse function of $l(x) = 3x - 6$.

Let $y = l(x)$. Then $y = 3x - 6$. Now swap x and y, that is:

$x = 3y - 6$ and solve for y.

$x = 3y - 6 \longrightarrow x + 6 = 3y$

$y = \frac{x+6}{3}$. So $\boxed{l^{-1}(x) = \frac{x+6}{3}}$

8. (15 points) Suppose $f(x) = \sqrt{x+2}$ and $g(x) = x^2 + 2$.

- Find the domain of $f(x)$.

Remember we can only take square root of nonnegative numbers:

$x + 2 \geq 0 \rightarrow \boxed{x \geq -2}$ or $\boxed{[-2, +\infty)}$

- Find the domain of $g(x)$.

No, even roots or division, thus domain is $\boxed{\mathbb{R}}$ or $\boxed{(-\infty, +\infty)}$

- Find $f(g(x))$.

$f(g(x)) = f(x^2+2) = \sqrt{(x^2+2)+2} = \boxed{\sqrt{x^2+4}}$

- Find $g(f(x))$.

$g(f(x)) = g(\sqrt{x+2}) = (\sqrt{x+2})^2 + 2 = x + 2 + 2 = \boxed{x+4}$

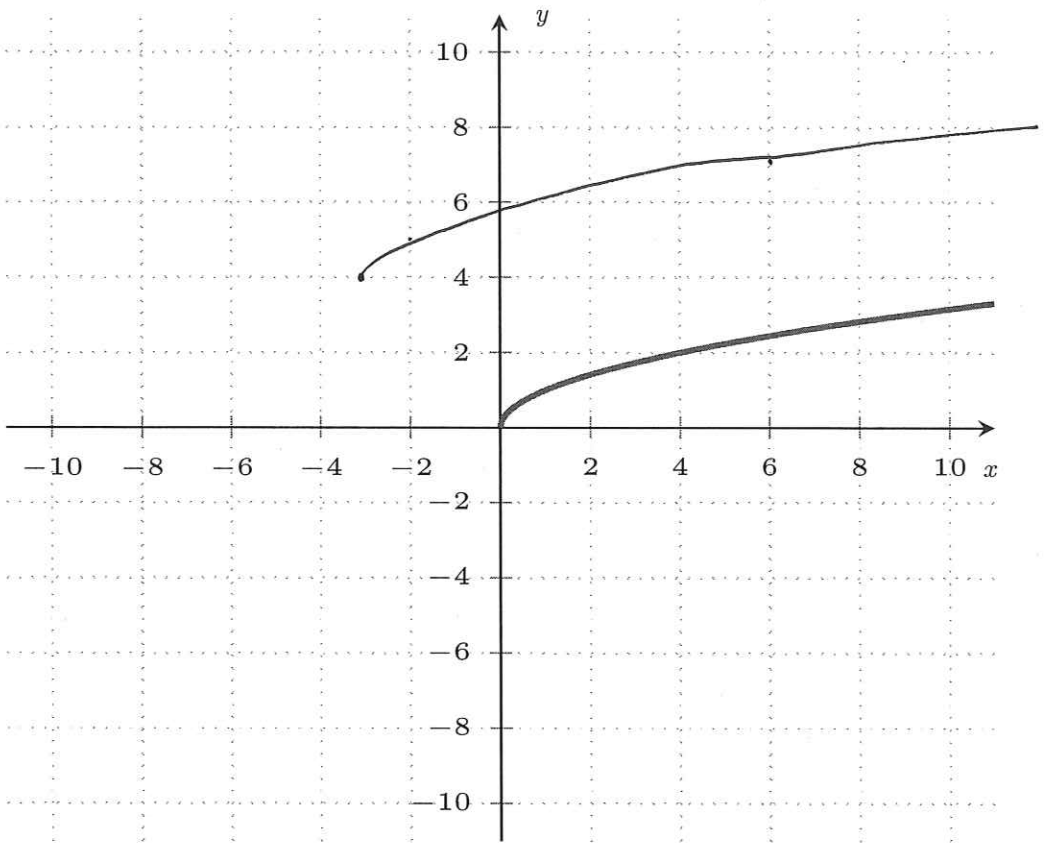
- Find $g(g(x))$.

$g(g(x)) = g(x^2+2) = (x^2+2)^2 + 2$

$= x^4 + 4x^2 + 4 + 2$

$= \boxed{x^4 + 4x^2 + 6}$.

9. (12 points) Below is a graph of $y = \sqrt{x}$. On the same axes sketch a graph of $y = \sqrt{x+3} + 4$.



10. (10 points) BONUS: Spell your professor name ☺

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Name: _____

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	15	8	10	10	10	10	15	12	10	110
Score:											