Review for Exam 2 - Part II

2   Functions

2.1   The Function Concept

2.1.1   Example

Does the equation $s^3 = 5t - 11$ define t as a function of s?

2.2   Function Notation

2.2.1   Example

Let $f(x) = x^3 - 4$. Find the following:

(a) What is $\dfrac{f(2) - f(y + 1)}{f(1)}$?

(b) What is $\dfrac{f(x + h) - f(x)}{h}$?

2.3 Piecewise-Defined Functions

2.3.1 Example

Let

$$f(x) = \begin{cases} x - 3 & \text{if } x < -2 \\ x^2 + 1 & \text{if } -2 \le x < 5 \\ \sqrt{x - 3} & \text{if } x > 5 \end{cases}$$

- Find $f(-5)$.

- Find $f(0)$.

- Find $f(5)$.

2.4 The Domain of a Function

2.4.1 Example

Find the domain of the following functions.

- $a(x) = x^2 - 2x + 7$.

- $b(x) = \dfrac{x - 1}{x}$.

- $c(x) = \sqrt{x - 2}$.

- $d(x) = \dfrac{x}{\sqrt{x - 1}}$.

2.5 Average Rates of Change

2.5.1 Example

Let $f(x) = x^3 - 4x + 3$. Find the average rate of change of $f(x)$ with respect to x as x changes from -2 to 2.

2.5.2 Example

Let $h(x) = 2x^2 - 1$. Find the average rate of change of $h(x)$ on the interval from x to $x + h$. Assume that $h \neq 0$. **Simplify**.

2.6 Operations on Functions

2.6.1 Example

Let $f(x) = \sqrt{x - 2}$ and $g(x) = x^2$.

- Find $(f + g)(6)$.

- Find $(fg)(x)$.

- Find $\left(\dfrac{f}{g}\right)(x)$ and its domain.

- Find $f(g(3))$.

- Find $g(f(x))$.

- Find $f(g(x))$.

2.7 Graph Transformations

2.7.1 Example

Let $g(x) = x^2$. Write $h(x)$ in terms of $g(x)$ and explain how you would transform the graph of g.

- $h(x) = (x-1)^2 + 3$.

- $h(x) = 3x^2 - 1$.

2.8 One-to-one Functions and Inverse Functions

2.8.1 Example

Let $f(x) = \dfrac{x-2}{5}$. Find $f^{-1}(x)$.

2.8.2 Example Challenging

Let $g(x) = x^2 + 4$. If g has an inverse function, find a formula for $g^{-1}(x)$. If g does not have an inverse function, can you think of a way to restrict the domain of g so that it does have an inverse function. (*Hint:* Restrict the domain of $g(x)$ so that $g(x)$ would become one-to-one function)