Review for Final Exam - Part II

2 Polynomials

2.1 Quadratic Functions

2.1.1 Example

Find the ~~maximum~~ value of the function $f(x) = 2x^2 + 11x - 4$.
minimum

min value of the function can be found by first finding the value of x at which this min value occurs:

$x = -\frac{b}{2a} = -\frac{11}{4}$ and then evaluating your function at it

$f\left(-\frac{11}{4}\right) = 2\cdot\left(-\frac{11}{4}\right)^2 + 11\left(\frac{-11}{4}\right) - 4 = 2\cdot\frac{121}{16} - \frac{121}{4} - 4 = \frac{121}{8} - \frac{121}{4} - 4 = \boxed{-\frac{153}{8}}$

2.1.2 Example

Find a quadratic function $f(x) = ax^2 + bx + c$ whose vertex is $(2, 3)$ and goes through the point $(5, 5)$.

Using the standard form of $f(x) = a(x-h)^2 + k$ with vertex $(2,3)$ gives

$f(x) = a(x-2)^2 + 3$.

Since $(5,5)$ is a point on the parabola, then $f(5) = 5$, or

$a(5-2)^2 + 3 = 5$

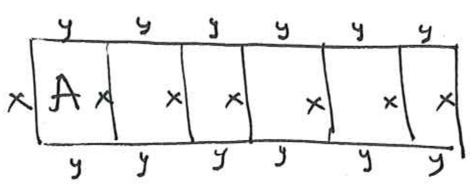
$a(3)^2 = 2$

$a = \frac{2}{9}$ Hence, $f(x) = \frac{2}{9}(x-2)^2 + 3 = \frac{2}{9}(x^2 - 4x + 4) + 3$

$= \frac{2}{9}x^2 - \frac{8}{9}x + \frac{8}{9} + 3 = \boxed{\frac{2}{9}x^2 - \frac{8}{9}x + \frac{35}{9}}$

2.1.3 Example

A farmer has 300 feet of fencing to construct 6 rectangular pens. What is the maximum possible area of all 6 pens?



The area, A, of each pen is $A = xy$.
The amount of fencing it takes to create the pens is given by $7x + 12y = 300$ or $y = \frac{300-7x}{12}$

So TA - total area $= 6A = 6xy = 6x\cdot\frac{300-7x}{12} = \frac{x}{2}(300-7x) = -\frac{7}{2}x^2 + 150x$

$x_{max} = \frac{-150}{2(-7/2)} = \frac{150}{7}$ $TA\left(\frac{150}{7}\right) = \frac{150}{7\cdot 2}\left(300 - 7\cdot\frac{150}{7}\right) = \frac{150^2}{14} = \frac{22{,}500}{14} \approx \boxed{1{,}607.14\,ft^2}$

2.2 Polynomial Division

2.2.1 Example

Find the quotient and the remainder.

$$\frac{4x^3 + x^2 - 6x + 8}{x+2}$$

$$
\begin{array}{r|l}
 & 4x^2 - 7x + 8 \\
\hline
x+2 & 4x^3 + x^2 - 6x + 8 \\
 & -\,(4x^3 + 8x^2) \\
\hline
 & -7x^2 - 6x + 8 \\
 & -\,(-7x^2 - 14x) \\
\hline
 & 8x + 8 \\
 & -\,(8x + 16) \\
\hline
 & -8
\end{array}
$$

Thus, the quotient is $\boxed{4x^2 - 7x + 8}$ and the remainder is $\boxed{-8}$.

2.2.2 Example

Find the quotient and the remainder.

$$\frac{6x^7 + 15x^3 + x - 8}{3x^2 + 7}$$

$$
\begin{array}{r|l}
 & 2x^5 - \frac{14}{3}x^3 + \frac{143}{9}x \\
\hline
3x^2 + 0\cdot x + 7 & 6x^7 + 0\cdot x^6 + 0\cdot x^5 + 0\cdot x^4 + 15x^3 + 0\cdot x^2 + x - 8 \\
 & -\,(6x^7 + 0x^6 + 14x^5) \\
\hline
 & -14x^5 + 0x^4 + 15x^3 + 0\cdot x^2 + x - 8 \\
 & -\,(-14x^5 + 0\cdot x^4 - \frac{98}{3}x^3) \\
\hline
 & \frac{143}{3}x^3 + 0x^2 + x - 8 \\
 & -\,(\frac{143}{3}x^3 + 0x^2 + \frac{1001}{9}x) \\
\hline
 & -\frac{992}{9}x - 8
\end{array}
$$

Thus, the quotient is $\boxed{2x^5 - 14/3\, x^3 + 143/9\, x}$ and the remainder is $\boxed{-\frac{992}{9}x - 8}$.

2.3 Finding Roots of Polynomials

2.3.1 Example

Let $P(x) = 5x^3 - 9x^2 + x - 14$. List all of the possible rational roots of $P(x)$ as given by the Rational Roots Theorem.

Factors of 14 and 5 :

14 : $\pm 1, \pm 2, \pm 7, \pm 14$

5 : $\pm 1, \pm 5$.

Possible roots are $\boxed{\pm 1, \pm 2, \pm 7, \pm 14, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{7}{5}, \pm \frac{14}{5}}$.

2.3.2 Example

Let $P(x) = 4x^4 - 7x + 6$. List all of the possible rational roots of $P(x)$ as given by the Rational Roots Theorem.

Factors of 6 and 4 :

6: $\pm 1, \pm 2, \pm 3, \pm 6$

4 : $\pm 1, \pm 2, \pm 4$

Possible roots are $\boxed{\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}}$

(Note that $\pm \frac{6}{4} = \pm \frac{3}{2}$, so it's already there).