Quiz # 2

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. (5 points) Find the center and radius of the circle whose equation is

$$x^2 - 6x + y^2 + 8y + 20 = 0.$$

Solution: To find the center and the radius of the circle we have to rewrite the above equation as $(x-h)^2 + (y-k)^2 = r^2$, where point (h,k) is a center of the circle and r its radius. Thus, we have to complete squares for both variables such as

$$\begin{aligned} x^2 - 6x + y^2 + 8y + 20 &= 0 \\ x^2 - 2\cdot 3 \cdot x + y^2 + 2 \cdot 4 \cdot y + 20 &= 0 \\ (x^2 - 2\cdot 3 \cdot x + 3^2) - 3^2 + (y^2 + 2\cdot 4 \cdot y + 4^2) - 4^2 + 20 &= 0 \\ (x-3)^2 + (y+4)^2 - 9 - 16 + 20 &= 0 \\ (x-3)^2 + (y+4)^2 &= 5 \text{ (i.e. } \left(\sqrt{5}\right)^2\text{)} \end{aligned}$$

Thus, the center of the circle is $(3,-4)$ and the radius is $\sqrt{5}$.

2. (5 points) Suppose $Q(3,0)$ is the midpoint of the line segment AB where A is the point $(1,-4)$. Find the point B. (Hint: Let B be the point (x_2, y_2); then use the definition of a midpoint to find x_2 and y_2).

Solution: Remember that the midpoint between any two points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ has the following coordinates:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

Let B be the point (x_2,y_2), and we know that A is the point $(1,-4)$, thus the midpoint of the line segment AB is

$$\left(\frac{1+x_2}{2}, \frac{-4+y_2}{2}\right) = (3,0).$$

since it's given to us that $Q(3,0)$ is the midpoint of the line segment AB.

Thus we have two equations:

$$\begin{aligned} \frac{1+x_2}{2} &= 3 \\ 1 + x_2 &= 6 \\ x_2 &= 5 \end{aligned} \qquad \begin{aligned} \frac{-4+y_2}{2} &= 0 \\ -4 + y_2 &= 0 \\ y_2 &= 4. \end{aligned}$$

Hence B is the point $(5,4)$.

3. (5 points) Find the x-intercepts and y-intercepts of the following equation:

$$x^2 - 4xy + 9y^2 = 1.$$

Solution: Remember that the y-value equals zero at an x-intercept and the x-value equals zero at a y-intercept. So having that in mind we get

$$\begin{aligned}
x-\text{intercept: } y = 0 \qquad & x^2 - 4 \cdot x \cdot 0 + 9 \cdot 0^2 = 1 \\
& x^2 = 1 \\
& x = \pm 1, \\
y-\text{intercept: } x = 0 \qquad & 0^2 - 4 \cdot 0 \cdot y + 9 \cdot y^2 = 1 \\
& 9y^2 = 1 \\
& y^2 = \frac{1}{9} \\
& y = \pm\sqrt{\frac{1}{9}} \\
& y = \pm\frac{1}{3}.
\end{aligned}$$

Thus, x-intercepts are: $(-1, 0)$ and $(1, 0)$ and y-intercepts are: $(0, -1/3)$ and $(0, 1/3)$.

Name: _____

Question:	1	2	3	Total
Points:	5	5	5	15
Score:				