Quiz #8

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit! Your answer to problem # 2 should be written in a clear and concise manner using a combination of complete sentences and symbolic expressions. An answer without explanation or that is poorly presented may not receive full credit.

1. (1 point) Find
$$\lim_{x\to 0} \frac{x^2}{1-\cos(x)}.$$

   A. 0

   B. $\frac{1}{2}$

   C. 1

   D. 2

   E. Does not exist.

2. (2 points) The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

Solution: Let $f(a,b) = a^2+b^2$, and we want to find the smallest possible value of it. Since $a+b=16$ where $a,b>0$, then $a = 16-b$ and the possible range of values for a and b are $[0,16]$. If we substitute $a = 16-b$ into $f(a,b)$, we get

$$f(a,b) = a^2+b^2 = (16-b)^2+b^2 = 256-32b+2b^2 = f(b).$$

Note: Similarly, we can solve for b and rewrite $f(a,b)$ as a function of a.

Now, let's find critical points of $f(b)$, i.e. take derivative and set it equals to zero:

$$f'(b) = -32+4b = 0.$$

So $b=8$ is the critical point of $f(b)$. Thus, we have to check $b = 0, 8, 16$ where $b=0$ and $b=16$ are end points of the interval. Hence

$$f(0) = 256 \qquad f(8) = 128 \qquad f(16) = 256.$$

From here we can conclude that $a = b = 8$ and the smallest possible value of $f(a,b)$ is

$$f(8,8) = 64+64 = \boxed{128}.$$

Name: _____

Question:	1	2	Total
Points:	1	2	3
Score:			