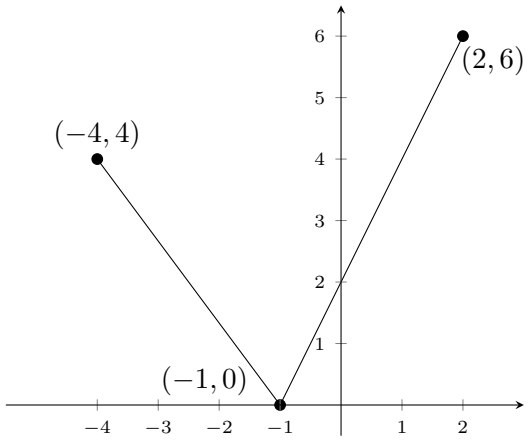
Written Assignment #1 - Solution

Let $f(x)$ be defined by the following three properties.

- The domain of $f(x)$ is $[-4, 2]$.
- The graph of $f(x)$ for x in $[-4, -1]$ is a straight line from the point $(-4, 4)$ to the point $(-1, 0)$.
- The graph of $f(x)$ for x in $[-1, 2]$ is a straight line from the point $(-1, 0)$ to the point $(2, 6)$.

1. (1 point) Sketch the graph of $f(x)$

Solution:



2. (2 points) Find the domain and range of $f(x)$. Briefly explain why your solution is correct.

Solution:

- Domain of $f(x)$: $[-4, 2]$.
- Range of $f(x)$: $[0, 6]$.

Explanation: Look at the graph above.

3. (2 points) Find $f(-2)$. Show your work.

Solution: Let's find an equation for the left segment of the function $f(x)$, i.e. the part that connects points $(-4, 4)$ and $(-1, 0)$. So

$$\text{slope} = \frac{0-4}{-1-(-4)} = -\frac{4}{3}$$

and using slope-point form we get that

$$f(x) = -\frac{4}{3}(x+4) + 4.$$

Thus

$$f(-2) = -\frac{4}{3}(-2+4) + 4 = -\frac{8}{3} + 4 = \frac{-8+12}{3} = \boxed{\frac{4}{3}}.$$

4. (3 points) Find the equation of the line through the point $(0, f(0))$ that is perpendicular to the graph $f(x)$.

Solution: Let's find an equation for the right segment of the function $f(x)$, i.e. the part that connects points $(-1, 0)$ and $(2, 6)$. So

$$\text{slope} = \frac{6-0}{2-(-1)} = \frac{6}{3} = 2$$

and using slope-point form we get that

$$f(x) = 2(x-2) + 6.$$

Thus

$$f(0) = 2(0-2) + 6 = -4 + 6 = 2.$$

Recall that the line that is perpendicular to the graph of $f(x)$ has a slope $-\frac{1}{2}$ i.e. negative reciprocal of the original slope m, in our case $m = 2$.

Again using slope-point form we get that

$$g(x) = -\frac{1}{2}(x-0) + 2 = \boxed{-\frac{1}{2}x + 2}.$$