Written Assignment #6 - Solution

1. (3 points) Evaluate

$$\lim_{x\to 0}\frac{e^x - x - 1}{\cos(x) - 1}$$

Solution: Apply L'Hôpital's rule twice.

$$\lim_{x\to 0}\frac{e^x - x - 1}{\cos(x) - 1} = \lim_{x\to 0}\frac{e^x - 1}{-\sin(x)} = \lim_{x\to 0}\frac{e^x}{-\cos(x)} = -\frac{e^0}{\cos(0)} = \boxed{-1}$$

2. A warehouse selling cement has to decide how often and in what quantities to reorder. It is cheaper, on average, to place large orders, because this reduces the ordering cost per unit. On the other hand, larger orders mean higher storage costs. the warehouse always reorders cement in the same quantity, q. The total weekly cost, C, or ordering and storage is given by

$$C = \frac{a}{q} + bq,$$

where a and b are positive constants.

(a) (1 point) Which of the terms a/q and bq represents the ordering cost and which represents the storage cost?

Solution:

$\frac{a}{q}$ - represents the ordering cost & bq - represents storage cost

(b) (4 points) What value of q gives the minimum total cost?

Solution: Let's find critical point(s), take derivative, set it equals to zero and solve for q. That's

$$C'(q) = -\frac{a}{q^2} + b = 0.$$

Then $q = \sqrt{\frac{a}{b}}$ is a critical point. Now, we have to use the second derivative test to classify this point. Find the second derivative of $C(q)$:

$$C''(q) = \frac{2a}{q^3}.$$

By plugging $q = \sqrt{\frac{a}{b}}$ into $C''(q)$ we have

$$C''\left(\sqrt{\frac{a}{b}}\right) = \frac{2a}{\left(\frac{a}{b}\right)^{3/2}} = \frac{2b^{3/2}}{a^{1/2}} > 0$$

since a and b are positive constants. Hence, we can conclude that $q = \sqrt{\frac{a}{b}}$ is a local min, but since $C''(q) > 0$ for all $q > 0$, then we can say that $q = \sqrt{\frac{a}{b}}$ is the absolute min.