Quiz #3

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit! Your answer to problem # 2 should be written in a clear and concise manner using a combination of complete sentences and symbolic expressions. An answer without explanation or that is poorly presented may not receive full credit.

1. (1 point) Suppose f and g are continuous functions such that $g(3) = 2$ and

$$\lim_{x\to 3} [4f(x) + f(x)g(x)] = 54.$$

Find $f(3)$.

A. $\frac{54}{7}$

B. 5

C. 0

D. 9

E. You cannot find it because $f(3)$ may not exist.

2. (2 points) If $f(x) = ax^2 + b$, find $f'(c)$ and use it to find an equation of the tangent line to the curve $y = ax^2 + b$ at the point $(c, ac^2 + b)$.

Solution: First, let find the derivative of $f'(c)$, that is

$$\begin{aligned} f'(c) &= \lim_{h\to 0} \frac{f(c+h) - f(c)}{h} \\ &= \lim_{h\to 0} \frac{a(c+h)^2 + b - (ac^2 + b)}{h} \\ &= \lim_{h\to 0} \frac{ac^2 + 2ach + ah^2 + b - ac^2 - b}{h} \\ &= \lim_{h\to 0} \frac{h(2ac + ah)}{h} = \lim_{h\to 0}(2ac + ah) = 2ac. \end{aligned}$$

Then using slope-point formula for the line, we get

$$\begin{aligned} y &= m(x - x_0) + y_0 \\ &= f'(c)(x - c) + f(c) \\ &= 2ac(x - c) + (ac^2 + b) \\ &= \boxed{2acx + b - ac^2}, \end{aligned}$$

which is the equation of the tangent line to the curve $y = ax^2 + b$ at the point $(c, ac^2 + b)$.

Name: _____

Question:	1	2	Total
Points:	1	2	3
Score:			