Quiz 6 — 10/12/17

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (1 point) By the ratio test, the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

A. converges absolutely
B. converges conditionally
C. diverges
D. the ratio test is inconclusive

Solution: Since $|a_{n+1}/a_n| \to 1$ as $n \to \infty$, the ratio test is inconclusive.

2. (1 point) The power series

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{3^n}$$

converges absolutely on the interval

A. $[-3, 3)$
B. $(-2, 4]$
C. $(-4, 2)$
D. $[-4, 2)$

Solution: $\left|\frac{a_{n+1}}{a_n}\right| \to \frac{|x+1|}{3}$ as $n \to \infty$, by the Ratio Test, we must have

$$\frac{|x+1|}{3} < 1 \Rightarrow |x+1| < 3 \Rightarrow -3 < x+1 < 3 \Rightarrow -4 < x < 2$$

. Checking the endpoints, the two series there do not converge, so the series converges absolutely on $(-4, 2)$.

3. (3 points) Determine whether the series

$$\sum_{n=1}^{\infty} \left(\frac{1-n}{1+3n} \right)^n$$

converges absolutely, converges conditionally, or diverges. Justify your answer using a convergence test.

Solution: Consider the limit

$$\lim_{n\to\infty} \sqrt[n]{\left| \frac{1-n}{1+3n} \right|^n} = \lim_{n\to\infty} \left| \frac{1-n}{1+3n} \right| = \frac{1}{3} < 1$$

Thus, the series converges absolutely by the Root Test.

(Assign 2 points for deciding to use the root test and correctly finding the limit, and the last point is for making the correct conclusion.)