

Quiz 1 — 09/08/16

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

1. Determine the partial fraction decomposition of the following fraction $\frac{x^2 + 6x - 6}{(x - 2)(x^2 + 1)}$.
(Absolutely no integration is involved in this question.)

Rewrite

$$\frac{x^2 + 6x - 6}{(x - 2)(x^2 + 1)} = \frac{A}{x - 2} + \frac{Bx + C}{x^2 + 1}$$

for some constants A , B , and C . Multiplying both sides of the above equation by $(x - 2)(x^2 + 1)$ and equating the coefficients, we have the following system of equations.

$$\begin{aligned}A + B &= 1 \\C - 2B &= 1 \\A - 2C &= -6\end{aligned}$$

So, $A = 2$, $B = 1$, and $C = -4$ and

$$\frac{x^2 + 6x - 6}{(x - 2)(x^2 + 1)} = \frac{2}{x - 2} + \frac{x - 4}{x^2 + 1}.$$

2. Find the following antiderivative: **Show your work!**

$$\int \sin^2 x \, dx.$$

$$\begin{aligned}\int \sin^2 x \, dx &= \int \left(\frac{1 + \cos(2x)}{2} \right) dx \\&= \frac{x}{2} + \frac{\sin 2x}{4} + C\end{aligned}$$