Quiz 5 — 10/05/17

Name: ____________________ Section and/or TA: ____________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ is convergent or divergent.

Solution: Consider $\int_1^{\infty} \frac{x}{x^2+1}\,dx$. Computing this improper integral we have

$$\int_1^{\infty} \frac{x}{x^2+1}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{x}{x^2+1}\,dx = \lim_{t\to\infty} \frac{1}{2}\ln(x^2+1)\Big|_1^t = +\infty$$

Since the integral diverges, the series also diverges.

2. Determine if the series $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$ is convergent or divergent.

Solution: Note that $\cos(n\pi) = (-1)^n$, so $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$ is an alternating series. Note that with $a_n = \frac{1}{\sqrt{n}}$, $\lim_{n\to\infty} a_n = 0$. We need to show that $a_{n+1} < a_n$. Since $\sqrt{n} < \sqrt{n+1}$, we have that $a_{n+1} = \frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}} = a_n$ and $\{a_n\}$ is decreasing. Thus, the series converges by the Alternating Series Test.