Quiz 8 — 11/22/16

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

1. Convert $(-1, \sqrt{3})$ from cartesian coordinates into polar coordinates with $r > 0$ and $0 \leq \theta < 2\pi$.

$r = x^2 + y^2 = (-1)^2 + (\sqrt{3})^2 = 4$, so $r = 2$.

If θ is the angle between the ray connecting the origin to the (rectangular) point $(-1, \sqrt{3})$ and the negative x axis, then $\tan(\theta) = \sqrt{3}$, and thus $\theta = \pi/3$. Therefore, the polar coordinates of $(-1, \sqrt{3})$ are given by $(2, 2\pi/3)$.

2. Let $r(\theta) = \cos(2\theta)$. Find the slope of the tangent line to r at $\theta = \pi/4$.

Recall

$$\frac{dy}{dx} = \frac{r' \cdot \sin(\theta) + r \cdot \cos(\theta)}{r' \cdot \cos(\theta) - r \cdot \sin(\theta)}$$

Using $r(\theta) = \cos(2\theta)$ yields:

$$\begin{aligned}
\frac{dy}{dx} &= \frac{-2\sin(2\theta) \cdot \sin(\theta) + \cos(2\theta) \cdot \cos(\theta)}{-2\sin(2\theta) \cdot \cos(\theta) - \cos(2\theta) \cdot \sin(\theta)} \\
&= \frac{-2\sin(\pi/2) \cdot \sin(\pi/4) + \cos(\pi/2) \cdot \cos(\pi/4)}{-2\sin(\pi/2) \cdot \cos(\pi/4) - \cos(\pi/2) \cdot \sin(\pi/4)} \\
&= 1
\end{aligned}$$