Quiz 3 — 09/29/16

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

1. Find the limit of the sequences $\{a_n\}_{n=1}^{\infty}$ defined as follows.

(a). $a_n = \dfrac{3n^2+5}{7n^2+n+3}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3n^2+5}{7n^2+n+3} \overset{\text{L'Hôpital}}{=} \lim_{n\to\infty} \frac{6n}{14n+1} \overset{\text{L'Hôpital}}{=} \lim_{n\to\infty} \frac{6}{14} = \frac{3}{7}$$

This sequence converges to $\frac{3}{7}$.

(b). $a_1 = 1, \quad a_n = a_{n-1} + 1$ for $n \geq 2$

This is the sequence

$$\begin{aligned} a_1 &= 1 \\ a_2 &= a_1 + 1 = 1 + 1 = 2 \\ a_3 &= a_2 + 1 = 2 + 1 = 3, \dots \end{aligned}$$

in general $a_n = n$ for all $n \geq 1$ and then $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} n = \infty$. The sequence diverges.

2. Are the following series convergent or divergent? When convergent find its sum. **Explain!**

(a). $\displaystyle\sum_{n=1}^{\infty} 5\frac{3^{n+1}}{7^n} = \sum_{n=1}^{\infty} 5\frac{3^2 3^{n-1}}{7^1 7^{n-1}} = \sum_{n=1}^{\infty} \frac{45}{7}\left(\frac{3}{7}\right)^{n-1}$ This is a geometric series of the form $\sum_{n=1}^{\infty} ar^{n-1}$ with $a = \frac{45}{7}$ and $r = \frac{3}{7}$. It is convergent since $|r| < 1$ and its sum is given by

$$S = \frac{a}{1-r} = \frac{\frac{45}{7}}{1-\frac{3}{7}} = \frac{\frac{45}{7}}{\frac{4}{7}} = \frac{45}{4}.$$

(b). $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+5}{7n^2+n+3}$

From problem 1(a) we have that

$$\lim_{n\to\infty} \frac{3n^2+5}{7n^2+n+3} = \frac{3}{7} \neq 0.$$

Since the test for divergence says that if $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ diverges, this is a divergent series.