

**Quiz 3 — 09/14/17**

Name: \_\_\_\_\_ Section and/or TA: \_\_\_\_\_

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (3 points)

- (a) (2 points) Use the Trapezoid rule with  $n = 5$  compute  $\int_1^2 \frac{1}{x} dx$ . Be sure to compute your answer to at least four decimal places.

**Solution:**  $\Delta x = 1/5$  and  $f(x) = 1/x$  so

$$\begin{aligned} T_5 &= \frac{1}{10} (f(1) + 2f(1.2) + 2f(1.4) + 2f(1.6) + 2f(1.8) + f(2)) \\ &= \frac{1}{10} \left( 1 + \frac{10}{6} + \frac{10}{7} + \frac{10}{8} + \frac{10}{9} + \frac{1}{2} \right) \\ &\simeq 0.69563 \text{ (1753/2520 as a fraction)} \end{aligned}$$

- (b) (1 point) What is the exact value of this integral? Compute the absolute error in the approximation you found in part (a). You may use a calculator to obtain a decimal value for the exact definite integral.

**Solution:**

$$\int_1^2 \frac{1}{x} dx = \ln(2) \simeq 0.69314718$$

so the absolute error is approximately

$$|0.69563 - 0.69315| \simeq 0.00248$$

2. (2 points) Determine whether the following improper integral converges or diverges. If it converges, compute the limit. Be sure to show your work!

$$\int_1^{\infty} \frac{1}{x^2 + 3x + 2} dx$$

**Solution:**

$$\begin{aligned}\lim_{t \rightarrow \infty} \int_1^t \frac{1}{x^2 + 3x + 2} dx &= \lim_{t \rightarrow \infty} \int \left( \frac{1}{x+1} - \frac{1}{x+2} \right) dx \\ &= \lim_{t \rightarrow \infty} \left[ \ln \left( \frac{x+1}{x+2} \right) \right]_1^t \\ &= \lim_{t \rightarrow \infty} \left[ \ln \left( \frac{t+1}{t+2} \right) - \ln \left( \frac{1+1}{1+2} \right) \right] \\ &= -\ln \frac{2}{3} = \ln \frac{3}{2}\end{aligned}$$