Quiz

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You MUST show your work to receive full credit!

1. (5 points) Compute the following limit:

$$\lim_{x\to\infty} \frac{7x^5+9x}{2x^6+x^5-x}.$$

Solution: We can eliminate the lower degree terms, so that we get

$$\lim_{x\to\infty} \frac{7x^5+9x}{2x^6+x^5-x} = \lim_{x\to\infty} \frac{7x^5}{2x^6} = \lim_{x\to\infty} \frac{7}{2x} = \frac{7}{\text{``}\infty\text{''}} = 0.$$

2. (5 points) Consider the function $f(x) = \sqrt{x+2}$. Calculate $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

Solution:

- First, let's find $f(x+h) = \sqrt{(x+h)+2} = \sqrt{x+h+2}$.
- Second, let's find $f(x+h) - f(x) = \sqrt{x+h+2} - \sqrt{x+2}$.
- Third, put the difference quotient together

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}.$$

- Fourth, take a limit (or at least try)

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}.$$

Oh no, we have a zero at the bottom, so can't take a limit yet, let's try to do something else.

- Fifth, multiply top and bottom by the conjugate of the top, then simplify and take a limit:

$$\begin{aligned}
\lim_{h\to 0} \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h} \left(\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}\right)
&= \lim_{h\to 0} \frac{\left(\sqrt{x+h+2}\right)^2-\left(\sqrt{x+2}\right)^2}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)} \\
&= \lim_{h\to 0} \frac{(x+h+2)-(x+2)}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)} \\
&= \lim_{h\to 0} \frac{x+h+2-x-2}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)} \\
&= \lim_{h\to 0} \frac{h}{h\left(\sqrt{x+h+2}+\sqrt{x+2}\right)} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h+2}+\sqrt{x+2}} \\
&= \frac{1}{\sqrt{x+2}+\sqrt{x+2}} \\
&= \frac{1}{2\sqrt{x+2}}.
\end{aligned}$$

Name: ____

Section (circle one): 021 022 023 024

Question:	1	2	Total
Points:	5	5	10
Score:			