Quiz

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You MUST show your work to receive full credit!

1. (5 points) Consider the function

$$f(x) = \begin{cases} x^2 & x \leq 1 \\ -4x + 5 & x > 1 \end{cases}$$

Find $\lim_{x \to 1} f(x)$, if it exists. Hint: you may find it helpful to sketch the graph.

Solution: For the limit $\lim_{x \to 1} f(x)$ to exist, we need to check that $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$ exist, and if they both exist, then we have to check whether $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$. If both of these conditions satisfied, then we can conclude that $\lim_{x \to 1} f(x)$ exists and equals $\lim_{x \to 1} f(x) = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$. So, for our function $f(x)$ we have

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2) = 1 \quad \text{and}$$
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} (-4x + 5) = 1.$$

Since $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 1$, then $\lim_{x \to 1} f(x)$ exists and $\lim_{x \to 1} f(x) = 1$.

2. (5 points) Consider the function

$$g(x) = \begin{cases} x^2 & x \leq 2 \\ A - 5x & x > 2 \end{cases}$$

What value of A will make $g(x)$ continuous at $x = 2$?

Solution: For the function $g(x)$ to be continuous at $x = 2$, $\lim_{x \to 2} g(x)$ has to exist and $\lim_{x \to 2} g(x) = g(2)$. But as we showed in the previous problem that $\lim_{x \to 2} g(x)$ would exist if and only if $\lim_{x \to 2^-} g(x) = \lim_{x \to 2^+} g(x)$. Clearly, $g(2) = (2)^2 = 4$. Now, we have

$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} (x^2) = 4 \quad \text{and}$$
$$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} (A - 5x) = A - 10.$$

So, we would have to set $4 = A - 10$ and solve it for A, clearly, $A = 14$.

Name: _____

Section (circle one): 021 022 023 024

Question:	1	2	Total
Points:	5	5	10
Score:			