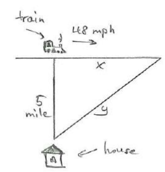
Quiz

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. (5 points) A train passes by a house at 48 miles per hour. Supposing the train has constant velocity and the shortest distance from the house to the tracks is 5 miles, how fast is the distance between the train and the house changing after 15 minutes?

Solution:

Suppose the distance that the train travels is x, then $\dfrac{dx}{dt} = 48$ mph which is exactly the velocity. The distance from the house to the train y satisfies $y^2 = 5^2 + x^2$ obtained by Pythagorean theorem. Taking the derivative with respect to t from both sides we have



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$$2y\frac{dy}{dt} = 0 + 2x\frac{dx}{dt} \quad \Longleftrightarrow \quad \frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}.$$

Since we are interested how fast distance between the train and the house changing after 15 minutes, then $x = 12$ miles (since train is moving at the constant speed 48 mph), and using $y^2 = 5^2 + x^2$ we can find that $y = \sqrt{25 + 12^2} = \sqrt{169} = 13$. So

$$\frac{dy}{dt} = \frac{12}{13} \cdot 48 = \frac{576}{13} \approx 44.3077 \, mph.$$

2. (5 points) Let $f(x) = 2x^3 + 3x^2 - 72x + 6$. Find the critical numbers, if any, and use them to find the maximum and minimum values of $f(x)$ on the interval [-5,5].

Solution: The problem asks us to find the critical numbers. Remember, critical numbers are the values of x where $f'(x) = 0$ or $f'(x)$ doesn't exist. Thus, we need to take derivative of $f(x)$ first, that is

$$f'(x) = 6x^2 + 6x - 72$$

and set it equal to zero,

$$f'(x) = 0 \quad \Longleftrightarrow \quad 6x^2 + 6x - 72 = 0.$$

Note that $6x^2 + 6x - 72 = 6(x-3)(x+4) = 0$. Thus $x = 3$ and $x = -4$ are the only critical numbers(points).

Finally, we have $f(-5) = 191$, $f(-4) = 214$, $f(3) = -129$, and $f(5) = -29$. Therefore, we can conclude that the minimum value of $f(x)$ occurs at $x = 3$ and the maximum value occurs at $x = -4$.

Name: ____

Section (circle one): 021 022 023 024

Question:	1	2	Total
Points:	5	5	10
Score:			