Quiz

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. (5 points) Let $f(x) = x^3 + 6x^2 - 15x + 3$. Find the critical numbers, if any, and use them to find the maximum and minimum values of $f(x)$ on the interval [-6,6].

Solution: The problem asks us to find the critical numbers. Remember, critical numbers are the values of x where $f'(x) = 0$ or $f'(x)$ doesn't exist. Thus, we need to take derivative of $f(x)$ first, that is

$$f'(x) = 3x^2 + 12x - 15$$

and set it equal to zero,

$$f'(x) = 0 \quad \Longleftrightarrow \quad 3x^2 + 12x - 15 = 0.$$

Note that $3x^2 + 12x - 15 = 3(x-1)(x+5) = 0$. Thus $x = 1$ and $x = -5$ are the only critical numbers(points).

Finally, we have $f(-6) = 93$, $f(-5) = 103$, $f(1) = -5$, and $f(6) = 345$. Therefore, we can conclude that the minimum value of $f(x)$ occurs at $x = 1$ and the maximum value occurs at $x = 6$ on the interval $[-6, 6]$.

2. (5 points) Find the critical points of the function $g(x) = 5xe^{(x^3+5)}$.

Solution: Similarly, to the previous problem we need to take derivative of our function $g(x)$ and look at the values where derivative is either 0 or doesn't exist. Using product and chain rules we get that the derivative of $g(x)$ is

$$g'(x) = 5e^{(x^3+5)} + 5xe^{(x^3+5)}(3x^2) = 5e^{(x^3+5)}(1 + 3x^3)$$

and set it equal to zero, $5e^{(x^3+5)}(1 + 3x^3) = 0$. Since $5e^{(x^3+5)} > 0$, thus we only need to consider $1 + 3x^3 = 0$, which have a root $x = -\sqrt[3]{\frac{1}{3}}$.

Thus, our function $g(x)$ has only one critical point at $x = -\sqrt[3]{\frac{1}{3}} = \frac{-3^{2/3}}{3}$.

Name: ____

Section (circle one): 021 022 023 024

Question:	1	2	Total
Points:	5	5	10
Score:			