Quiz

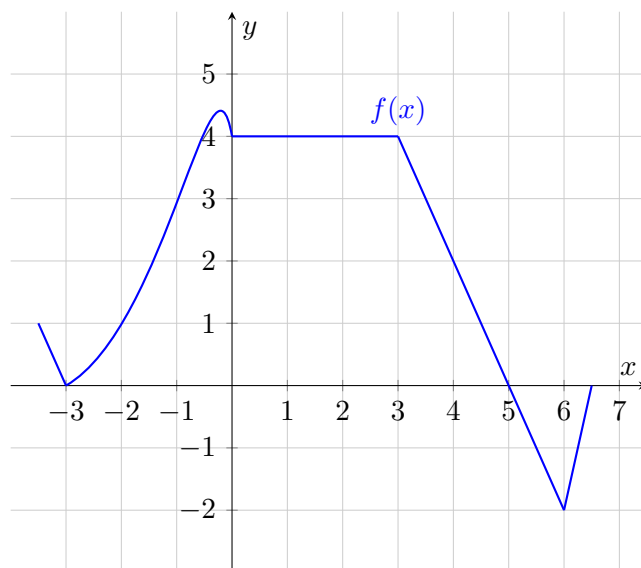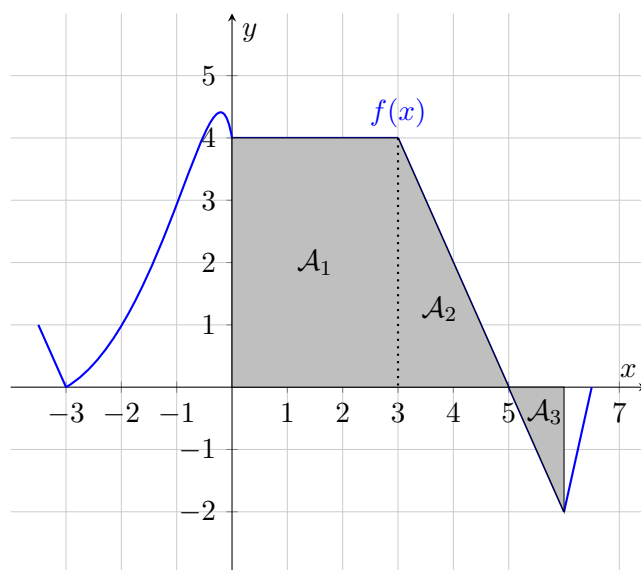**Directions:** Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. (5 points) Given the graph $y = f(x)$ below, compute the integral $\int_0^6 f(x)dx$ using geometry.

Solution:

The region between the x-axis and the function $f(x)$ forms a rectangle with a base of length 3 and height of length 4 (i.e. $\mathcal{A}_1$ see picture above), a triangle with a base of length 2 and

a height of length 4 (from the x values of 3 to 5) (i.e. $\mathcal{A}_2$ see picture above), and another triangle with a base of length 1 and a height of length 4 (from the x values of 5 to 6) (i.e. $\mathcal{A}_3$ see picture above). But last triangle lies below x-axis, so we would need to subtract it. Therefore,

$$\int_0^6 f(x)dx = \mathcal{A}_1 + \mathcal{A}_2 - \mathcal{A}_3 = (3)(4) + \frac{1}{2}(2)(4) - \frac{1}{2}(1)(2) = 15.$$

2. (5 points) Suppose $\int_1^6 f(x)\ dx = -3$. Find $\int_6^1 (2f(x) - 3)\ dx$.

Solution: To find that integral we will use the following three properties

$$\int_a^b (f(x) \pm g(x))\ dx = \int_a^b f(x)\ dx \pm \int_a^b g(x)\ dx,$$

$$\int_a^b f(x)\ dx = - \int_b^a f(x)\ dx,$$

and

$$\int_a^b cf(x)\ dx = c \int_a^b f(x)\ dx,\ \text{where } c \in \mathbb{R}.$$

Thus, using them we obtain

$$\int_6^1 (2f(x) - 3)\ dx = - \int_1^6 (2f(x) - 3)\ dx$$
$$= \int_1^6 (-2f(x) + 3)\ dx$$
$$= \int_1^6 -2f(x)\ dx + \int_1^6 3\ dx$$
$$= -2 \int_1^6 f(x)\ dx + \int_1^6 3\ dx$$
$$= -2(-3) + 3(5) = 21.$$

Name: _____

Section (circle one):   021   022   023   024

Question:	1	2	Total
Points:	5	5	10
Score:			