

Quiz

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. Consider the function

$$f(x) = \sqrt{x-5}.$$

- (a) (5 points) Use the limit definition of derivative to find $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$.

Solution:

- First, let's find $f(x+h) = \sqrt{(x+h)-5} = \sqrt{x+h-5}$.
- Second, let's find $f(x+h) - f(x) = \sqrt{x+h-5} - \sqrt{x-5}$.
- Third, put the difference quotient together

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h}.$$

- Fourth, take a limit (or at least try)

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h}.$$

Oh no, we have a zero at the bottom, so can't take a limit yet, let's try to do something else.

- Fifth, multiply top and bottom by the conjugate of the top, then simplify and take a limit:

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} &= \left(\frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \right) \left(\frac{\sqrt{x+h-5} + \sqrt{x-5}}{\sqrt{x+h-5} + \sqrt{x-5}} \right) \\ &= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h-5})^2 - (\sqrt{x-5})^2}{h(\sqrt{x+h-5} + \sqrt{x-5})} \\ &= \lim_{h \rightarrow 0} \frac{(x+h-5) - (x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})} \\ &= \lim_{h \rightarrow 0} \frac{x+h-5-x+5}{h(\sqrt{x+h-5} + \sqrt{x-5})} \\ &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h-5} + \sqrt{x-5})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h-5} + \sqrt{x-5}} \\ &= \frac{1}{\sqrt{x-5} + \sqrt{x-5}} \\ &= \frac{1}{2\sqrt{x-5}}. \end{aligned}$$

- (b) (5 points) Find the derivative $f'(x)$ by using the power rule.

Solution: We know that $f(x) = \sqrt{x-5}$ can be also rewritten as $f(x) = (x-5)^{\frac{1}{2}}$. Now, let us use the power rule to find $f'(x)$. According to the *Power rule*, power must go down and then we subtract 1 from the original power, in other words,

$$f'(x) = \frac{1}{2}(x-5)^{(\frac{1}{2}-1)} = \frac{1}{2}(x-5)^{-\frac{1}{2}} = \frac{1}{2(x-5)^{\frac{1}{2}}} = \frac{1}{2\sqrt{x-5}}.$$

(Note: We get the same answer as in part (a). Coincidence?...)

Name: _____

Section (circle one): 021 022 023 024

Question:	1	Total
Points:	10	10
Score:		