Quiz #6

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. (5 points) Use the limit definition of the derivative to find $f'(4)$ for the function

$$\frac{1}{x+1}.$$

Solution: Recall the definition of derivative of function f at $x = c$:

$$f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c)}{h}.$$

Having this in mind we have,

$$\begin{aligned}
f'(4) &= \lim_{h\to 0} \frac{f(4+h) - f(4)}{h} = \lim_{h\to 0} \frac{\frac{1}{(4+h)+1} - \frac{1}{4+1}}{h} = \lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} \\
&= \lim_{h\to 0} \frac{\left(\frac{5-(5+h)}{5(5+h)}\right)}{h} = \lim_{h\to 0} \frac{5-5-h}{5h(5+h)} = \lim_{h\to 0} \frac{-h}{5h(5+h)} \\
&= \lim_{h\to 0} \frac{-1}{5(5+h)} = \frac{-1}{5(5+0)} = \boxed{-\frac{1}{25}}.
\end{aligned}$$

2. (5 points) Let $g(x) = -3x^2 + x + 1$. Find an equation for the tangent line to g at the value $x = 2$.

Solution: First, we are going to find the slope of the tangent line. **Remember:** slope of the tangent line to the function g at the value $x = 2$, is the derivative of g evaluated at $x = 2$. Therefore, let's find derivative of g and evaluate it at $x = 2$, that is

$$g'(x) = -6x + 1, \quad \text{and} \quad g'(2) = -6(2) + 1 = \mathbf{-11}.$$

Now, we know the slope but we still need a point, which is easy to find since we know the x component of the point (i.e. $x = 2$) and we know the equation of the function g, so let's find the y component (i.e. y) that is

$$g(2) = -3(2)^2 + (2) + 1 = -9.$$

Thus, our point is **(2,-9)** and we can use the slope-point equation to find the formula for the tangent line to g at the value $x = 2$. So

$$y = -11x + b \quad \rightsquigarrow \quad -9 = -11(2) + b \quad \rightsquigarrow \quad b = 13.$$

Concluding that the equation for the tangent line to g at the value $x = 2$ is

$$\boxed{y = -11x + 13}.$$

Name: ____

Section (circle one): 003 004

Question:	1	2	Total
Points:	5	5	10
Score:			