(3)

Find all the x-coordinates in the increasing order of the points on the curve $x^2y^2 + xy = 2$ where the slope of the tangent is -1.

Solution

$$x^2y^2 + xy = 2$$

$$2x \cdot y^2 + \underline{x^2 \cdot 2y \frac{dy}{dx}} + y + \underline{x \cdot \frac{dy}{dx}} = 0$$

$$\frac{dy}{dx}\left(2x^2y + x\right) = -y - 2xy^2$$

$$\frac{dy}{dx} = \frac{-(y+2xy^2)}{2x^2y+x} = \frac{-y(1+2xy)}{x(2xy+1)} = -y/x$$

Set $\frac{dy}{dx} = -1$, so $-y/x = -1 \iff y = x$

Now substitute that into original equation.

$$x^2 \cdot x^2 + x \cdot x = 2$$

$$x^4 + x^2 - 2 = 0$$

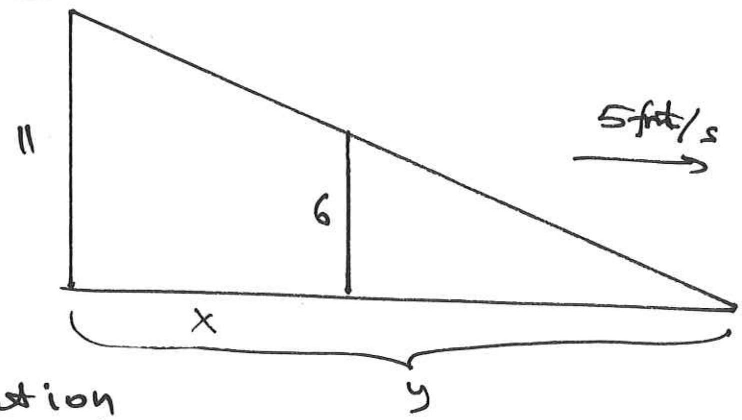
Let $z = x^2$, then $z^2 + z - 2 = 0$

$$(z-1)(z+2) = 0$$

Thus, $z = 1$ or $\cancel{z = -2}$ (since $z = x^2 \geq 0$)

Hence, $z = x^2 = 1 \implies \boxed{x = -1}$ and $\boxed{x = 1}$

(16)



Solution

We are going to use similar triangles.

$$\frac{11}{y} = \frac{6}{y-x} \quad \text{or} \quad 11y - 11x = 6y$$

$$5y = 11x$$

$$y = \tfrac{11}{5}x$$

$$\frac{dy}{dt} = \tfrac{11}{5} \cdot \frac{dx}{dt}$$

$$\frac{dy}{dt}(40) = \tfrac{11}{5} \cdot 5 = \boxed{11}\ ft/s.$$