1. **(Problem # 89, p. 79)**

   Consider the sequence recursively defined by the relation

   $$a_{n+1} = \frac{a_n}{1+a_n} \quad a_0 = 1.$$

   Compute a_n for $n = 1, 2, \ldots, 5$.

   Solution: We have to compute a_1 through a_5 using given recursion relation and initial condition $a_0 = 1$:

   $$\begin{aligned}
   a_0 &= 1 \\
   a_1 &= \frac{a_0}{1+a_0} = \frac{1}{1+1} = \boxed{\frac{1}{2}} \\
   a_2 &= \frac{a_1}{1+a_1} = \frac{\frac{1}{2}}{1+\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \boxed{\frac{1}{3}} \\
   a_3 &= \frac{a_2}{1+a_2} = \frac{\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{4}{3}} = \boxed{\frac{1}{4}} \\
   a_4 &= \frac{a_3}{1+a_3} = \frac{\frac{1}{4}}{1+\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{5}{4}} = \boxed{\frac{1}{5}} \\
   a_5 &= \frac{a_4}{1+a_4} = \frac{\frac{1}{5}}{1+\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{6}{5}} = \boxed{\frac{1}{6}}
   \end{aligned}$$

2. **(Problem # 101, p. 79)**

Consider the sequence recursively defined by the relation

$$a_{n+1} = \sqrt{5a_n}$$

Find all fixed points of $\{a_n\}$.

Solution: Notice that $a_{n+1} = f(a_n)$ where $f(x) = \sqrt{x})$

To find the fixed points, we need to solve for a in:

$$a = \sqrt{5a}.$$

That's

$$\begin{aligned} a = \sqrt{5a} \quad &\iff \quad a^2 = 5a \\ &\implies \quad a^2 - 5a = 0 \\ &\iff \quad a(a-5) = 0 \\ &\iff \quad a = 0 \text{ or } a = 5. \end{aligned}$$

Thus, there are two fixed points:

$$\boxed{a_1 = 0} \quad \text{and} \quad \boxed{a_2 = 5}.$$

3. **(Problem # 108, p. 79)**

Consider the sequence recursively defined by the relation

$$a_{n+1} = 2a_n(1 - a_n) \qquad a_0 = 0$$

and assume that $\lim_{n\to\infty} a_n$ exists.

Find all fixed points of $\{a_n\}$, and use a table or other reasoning to guess which fixed point is the limiting value for the given initial condition.

Solution: First, let's find the fixed points, we need to solve for a in:

$$a = 2a(1 - a).$$

That's

$$\begin{aligned} a = 2a(1-a) \quad &\iff \quad a = 2a - 2a^2 \\ &\implies \quad 2a^2 - a = 0 \\ &\iff \quad a(2a-1) = 0 \\ &\iff \quad a = 0 \text{ or } a = \frac{1}{2}. \end{aligned}$$

Thus, there are two fixed points:

$$\boxed{a_1 = 0} \quad \text{and} \quad \boxed{a_2 = \frac{1}{2}}.$$

Now, let's investigate on $\lim_{n\to\infty} a_n$. Let's work out a few terms of this sequence:

$$\begin{aligned} a_0 &= 0 \\ a_1 &= 2a_0(1 - a_0) = 2(0)(1-0) = 0 \\ a_2 &= 2a_1(1 - a_1) = 2(0)(1-0) = 0 \\ &\vdots \end{aligned}$$

continuing the same process, we will realize that $a_n = 0$ for $n = 0, 1, 2, 3, \ldots$, in other words, it's a constant sequence of zeros. Thus,

$$\boxed{\lim_{n\to\infty} a_n = 0}.$$