1. **(Problem # 43, p. 53)**

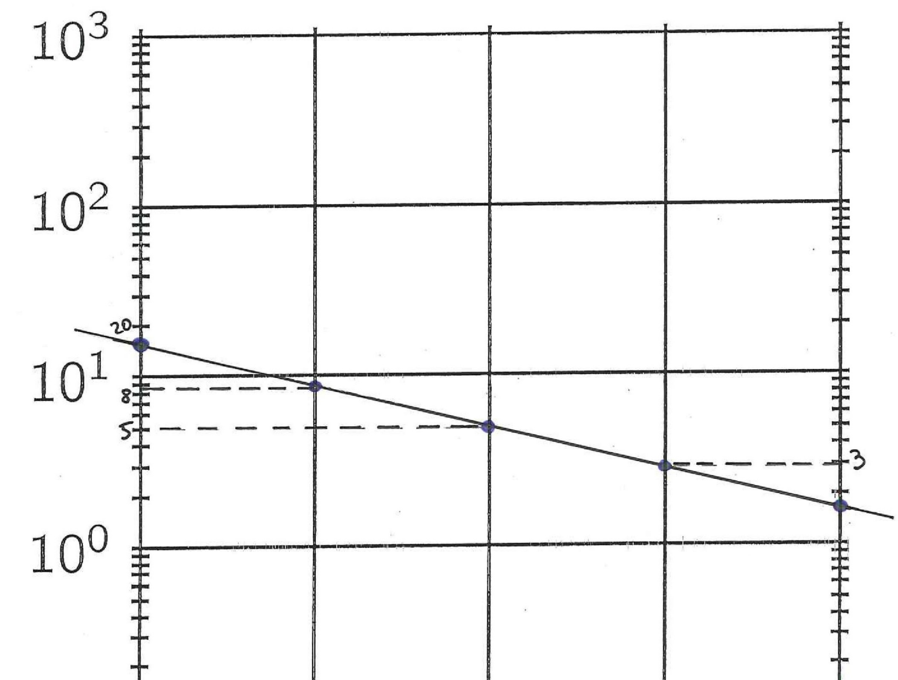
   When $\log y$ is graphed as a function of x, a straight line results. Graph the straight line given by the following two points
$$(x_1, y_1) = (0, 5) \qquad (x_2, y_2) = (3, 1)$$
   on a log-linear plot.

   (**Note:** The original $x - y$ coordinates are given.)

Solution: We are going to use the second method. That is

$$\begin{array}{ccc} (x_1, y_1) = (0, 5) & \text{and} & (x_2, y_2) = (3, 1) \\ \downarrow & & \downarrow \\ (x_1, Y_1) = (0, \log(5)) & \text{and} & (x_2, Y_2) = (3, \log(1)) \\ \| & & \| \\ (x_1, Y_1) = (0, \log(5)) & \text{and} & (x_2, Y_2) = (3, 0). \end{array}$$

Now, let's find the slope:
$$m = \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{0 - \log(5)}{3 - 0} = -\frac{1}{3} \cdot \log(5).$$
Since we know points on the line and the slope of the line, we can use point-slope equation to find the desire formula. We are going to use the point $(x_1, Y_1) = (0, \log(5))$, so
$$Y - Y_1 = m(x - x_1) \quad \rightsquigarrow \quad Y - \log(5) = -\frac{1}{3} \cdot \log(5)(x - 0) \quad \rightsquigarrow \quad \log\left(\frac{y}{5}\right) = \log\left(5^{-\frac{x}{3}}\right)$$
$$\rightsquigarrow \quad \frac{y}{5} = 5^{-\frac{x}{3}} \quad \rightsquigarrow \quad \boxed{y = 5 \cdot 5^{-\frac{x}{3}} \approx 5 \cdot (0.58)^x}.$$



2. **(Problem # 47, p. 53)**

Consider the relationship $y = 3 \cdot 10^{-2x}$ between the quantities x and y.

Use a logarithmic transformation to find a linear relationship of the form

$$Y = mx + b$$

between the given quantities.

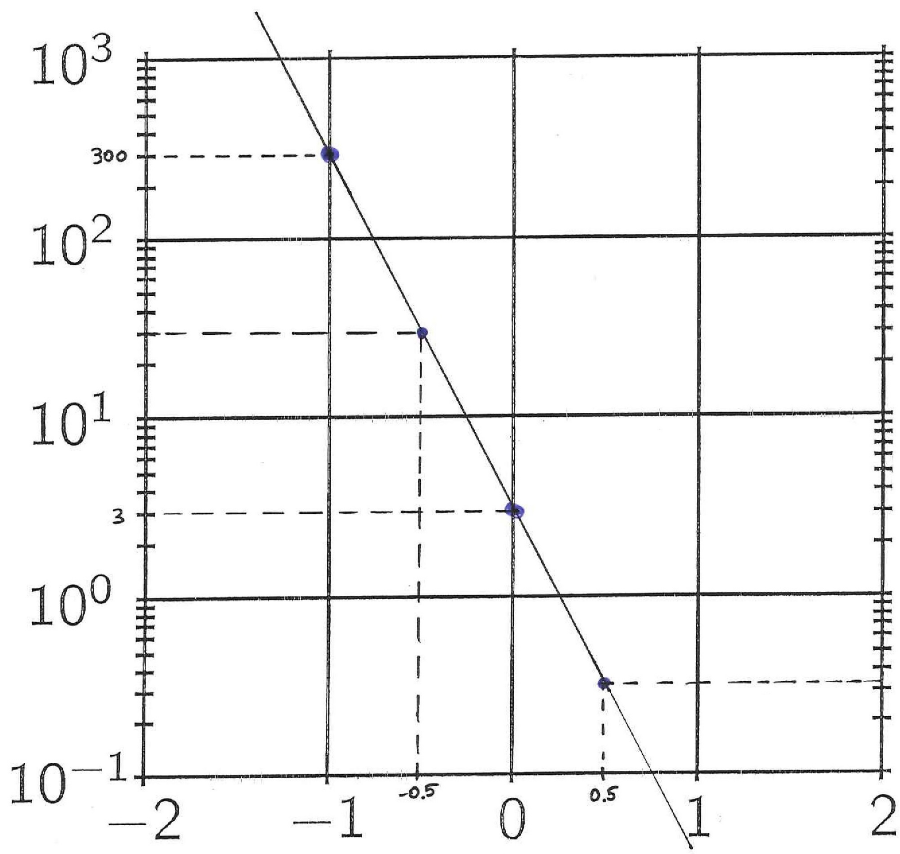
Graph the resulting linear relationship on a log-linear plot.

Solution: Consider the following logarithmic transformation:

$$\begin{aligned} y = 3 \cdot 10^{-2x} &\rightsquigarrow \log(y) = \log\left(3 \cdot 10^{-2x}\right) \\ &\rightsquigarrow \log(y) = \log(3) + \log\left(10^{-2x}\right) \\ &\rightsquigarrow \log(y) = \log(3) - 2\log(10)x \\ &\rightsquigarrow Y = \log(3) - 2x. \end{aligned}$$

Thus

$$\boxed{Y = \log(3) - 2x} \qquad \boxed{m = -2} \qquad \boxed{b = \log(3)}.$$



3. **(Problem # 57, p. 53)**

When $\log y$ is graphed as a function of $\log x$, a straight line results. Graph the straight line given by the following two points

$$(x_1, y_1) = (4, 2) \qquad (x_2, y_2) = (8, 8)$$

on a log-log plot.

(**Note:** The original $x - y$ coordinates are given.)

Solution: We are going to use the second method. That is

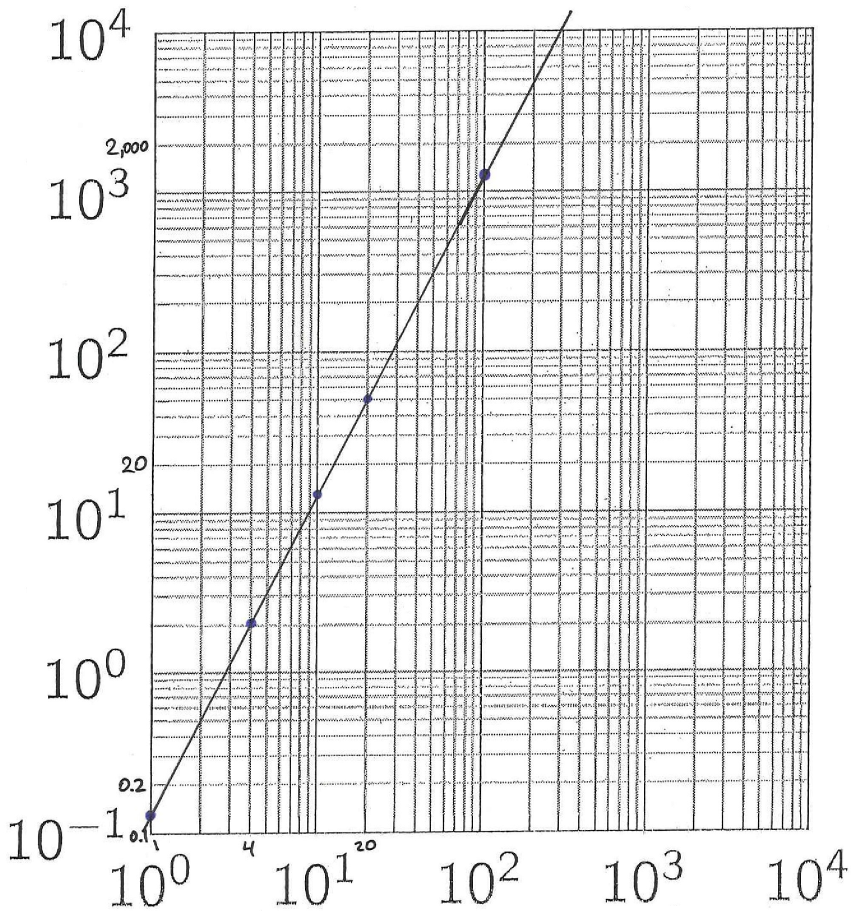
$$\begin{array}{ccc} (x_1, y_1) = (4, 2) & \text{and} & (x_2, y_2) = (8, 8) \\ \downarrow & & \downarrow \\ (X_1, Y_1) = (\log(4), \log(2)) & \text{and} & (X_2, Y_2) = (\log(8), \log(8)). \end{array}$$

Now, let's find the slope:

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{\log(8) - \log(2)}{\log(8) - \log(4)} = \frac{\log\left(\frac{8}{2}\right)}{\log\left(\frac{8}{4}\right)} = \frac{\log(4)}{\log(2)}.$$

Since we know points on the line and the slope of the line, we can use point-slope equation to find the desire formula. We are going to use the point $(X_1, Y_1) = (\log(4), \log(2))$, so

$$\begin{aligned} Y - Y_1 = m(X - X_1) \quad &\rightsquigarrow \quad Y - \log(2) = \frac{\log(4)}{\log(2)}(X - \log(4)) \\ &\rightsquigarrow \quad \log(y) - \log(2) = \frac{\log(4)}{\log(2)}(\log(x) - \log(4)) \\ &\rightsquigarrow \quad \log\left(\frac{y}{2}\right) = \frac{\log(4)}{\log(2)} \log\left(\frac{x}{4}\right) \\ &\rightsquigarrow \quad \log\left(\frac{y}{2}\right) = \log_2(4) \log\left(\frac{x}{4}\right) \\ &\rightsquigarrow \quad \log\left(\frac{y}{2}\right) = 2\log\left(\frac{x}{4}\right) \\ &\rightsquigarrow \quad \log\left(\frac{y}{2}\right) = \log\left(\left(\frac{x}{4}\right)^2\right) \\ &\rightsquigarrow \quad \frac{y}{2} = \left(\frac{x}{4}\right)^2 \\ &\rightsquigarrow \quad y = 2 \cdot \frac{x^2}{16} \\ &\rightsquigarrow \quad \boxed{y = \frac{x^2}{8}}. \end{aligned}$$


10^4
2,000
10^3
10^2
20
10^1
10^0
0.2
10^{-1}
0.1
1
4
20
10^0
10^1
10^2
10^3
10^4

4. **(Problem # 59, p. 53)**

Consider the relationship $y = 2 \cdot x^5$ between the quantities x and y. Use a logarithmic transformation to find a linear relationship of the form

$$Y = mX + b$$

between the given quantities.

Graph the resulting linear relationship on a log-log plot.

Solution: Consider the following logarithmic transformation:

$$\begin{aligned} y = 2 \cdot x^5 &\leadsto \log(y) = \log\left(2 \cdot x^5\right) \\ &\leadsto \log(y) = \log(2) + \log\left(x^5\right) \\ &\leadsto \log(y) = \log(2) + 5\log(x) \\ &\leadsto Y = \log(2) + 5X. \end{aligned}$$

Thus

$$\boxed{Y = \log(2) + 5X} \qquad \boxed{m = 5} \qquad \boxed{X = \log(x)} \qquad \boxed{b = \log(2)}.$$