Quiz #10

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. Consider the difference equation:
$$x_{n+1} = x_n^2.$$
   (a) (3 points) Find the fixed point(s) of the difference equation.

Solution: Assume that $\lim_{n\to\infty} x_n = \hat{x}$. To find the fixed points we need to solve the following equation
$$\hat{x} = \hat{x}^2.$$
So using factoring we get
$$\hat{x} = \hat{x}^2 \quad \Longleftrightarrow \quad \hat{x}(\hat{x}-1) = 0,$$
thus $\hat{x} = 0$ or $\hat{x} = 1$, two fixed points.

   (b) (2 points) Identify the fixed point(s) as locally stable or unstable.

Solution: Notice that our difference equation has the form $x_{n+1} = f(x_n)$ where $f(x) = x^2$. Thus using stability criterion we can determine where or not our fixed points are stable.
So $f'(x) = 2x$ and
$$\begin{aligned} \left|f'(0)\right| = 2(0) = 0 < 1 \quad &\Longrightarrow \quad \hat{x} = 0 \text{ is locally stable fixed point,} \\ \left|f'(1)\right| = 2(1) = 2 > 1 \quad &\Longrightarrow \quad \hat{x} = 1 \text{ is locally unstable fixed point.} \end{aligned}$$

2. (5 points) Find $f(x)$ given that
$$f'(x) = x^2 + e^{2x}, \quad f(0) = 0.$$

Solution: So the anti-derivative of $f'(x)$ is
$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}e^{2x} + C$$
Don't forget about C. Then let's using initial condition $f(0) = 0$ to determine C, that is
$$0 = f(0) = 0 + \frac{1}{2} + C,$$
thus $C = -\frac{1}{2}$. Finally,
$$\boxed{f(x) = \frac{1}{3}x^3 + \frac{1}{2}e^{2x} - \frac{1}{2}}.$$

Name: ______________________________

Section (circle one): 003 004

Question:	1	2	Total
Points:	5	5	10
Score:			