Quiz #2

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

1. (5 points) Solve for x, $5^{2x-5} = 2^{x+6}$.

Solution: To solve for x we will need to take ln of both sides and then simplify. That is

$$\begin{aligned} 5^{2x-5} &= 2^{x+6} \\ \ln(5^{2x-5}) &= \ln(2^{x+6}) \\ (2x-5)\ln 5 &= (x+6)\ln 2 \\ 2x\ln 5 - 5\ln 5 &= x\ln 2 + 6\ln 2 \\ 2x\ln 5 - x\ln 2 &= 6\ln 2 + 5\ln 5 \\ x(2\ln 5 - \ln 2) &= 6\ln 2 + 5\ln 5 \\ x &= \boxed{\frac{6\ln 2 + 5\ln 5}{2\ln 5 - \ln 2}}. \end{aligned}$$

2. Assume that the number of bacteria follows an exponential growth model: $P(t) = P_0 e^{kt}$. The count in the bacteria culture was 100 after 10 minutes and 1,000 after 30 minutes.

(a) (3 points) What was the initial size of the culture?

Solution: Since our model looks like $P(t) = P_0 e^{kt}$, where P_0 represents the initial size of the culture, and we know that the count in the bacteria culture was 100 after 10 minutes and 1,000 after 30 minutes, then we can set up the following system of equations with two unknowns P_0 and k, and then solve for P_0. Then

$$100 = p(10) = P_0 e^{k10} \quad \text{and} \quad 1,000 = p(30) = P_0 e^{k30}$$

$$P_0 = \frac{100}{e^{k10}} \quad \rightsquigarrow \quad 1,000 = \frac{100}{e^{k10}} e^{k30} \quad \rightsquigarrow \quad 10 = e^{k20} \quad \rightsquigarrow \quad \ln 10 = k20$$

$$k = \frac{\ln 10}{20}$$

$$P_0 = \frac{100}{e^{\frac{\ln 10}{20}10}} = \frac{100}{e^{\frac{1}{2}\ln 10}} = \frac{100}{\sqrt{10}} = \boxed{10\sqrt{10}} \approx \boxed{31.62278} \text{ bacteria.}$$

(b) (2 points) Find the population after 60 minutes.

Solution: Simply need to evaluate our model at $P(t)$ at $t = 60$. That is,

$$P(60) = 10\sqrt{10} e^{\frac{\ln 10}{20} \cdot 60} = \boxed{10,000\sqrt{10}} \approx \boxed{31,622.8} \text{ bacteria.}$$

Name: ____

Section (circle one): 003 004

Question:	1	2	Total
Points:	5	5	10
Score:			