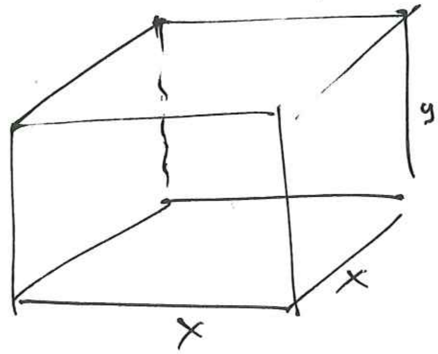
①

no top



Volume $x^2 \cdot y = 8{,}000 \text{ cm}^3$

Hence $\boxed{y = \frac{8{,}000}{x^2}}$.

The box is open, so no top and surface are then is

$$S(x) = \underbrace{4xy}_{4\text{ sides}} + \underbrace{x^2}_{\text{base}} = 4x \cdot \frac{8{,}000}{x^2} + x^2 = \boxed{\frac{32{,}000}{x} + x^2}.$$

No constraints on x other than $\boxed{0 < x < \infty}$.

This forces $S > 0$. And

$$S'(x) = -\frac{32{,}000}{x^2} + 2x = \boxed{\frac{2x^3 - 32{,}000}{x^2}}.$$

$$S'(x) = 0 \iff 2x^3 - 32{,}000 = 0 \iff x^3 = 16{,}000$$

$$\iff x = \sqrt[3]{16{,}000} \approx \boxed{25.198421 \text{ cm}}.$$

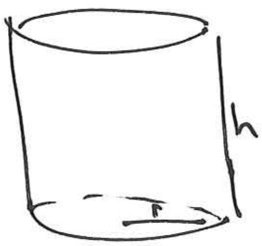
sign of $S'(x)$

Length of base $= x \approx 25.1984$ cm

Height $= \frac{8{,}000}{25.1984^2} \approx 12.5992105$ cm

Area of Material $= S \approx 1{,}904.8812624 \text{ cm}^2$

② Similar problem to #1 (for more details look at #1)



$$\text{Volume} = \pi r^2 h = 2{,}744 \text{ cm}^3$$

$$h = \frac{2{,}744}{\pi r^2}, \quad \text{constraint } r > 0.$$

$$\text{Surface Area} = \underbrace{2\pi r h}_{\text{sides}} + \underbrace{2\pi r^2}_{\substack{\text{top} \\ + \text{bottom}}}$$

$$= 2\pi \cancel{r} \cdot \frac{2{,}744}{\cancel{\pi} r^{\cancel{2}}} + 2\pi r^2 = \frac{5{,}488}{r} + 2\pi r^2 = S(r)$$

$$S'(r) = -\frac{5{,}488}{r^2} + 4\pi r = \frac{4\pi r^3 - 5{,}488}{r^2}$$

$$S'(r) = 0 \iff 4\pi r^3 - 5{,}488 = 0$$

$$r = \sqrt[3]{\frac{5{,}488}{4\pi}} \approx$$

r - radius of can $\approx$ 7.5869658 cm

h - heigh $\approx$ 15.17393 cm

$S(r)$ - Area of material $\approx$ 1,085.0188474 cm^2.

(3) All you have to do is to find the critical number of V.

So take derivative of V with respect to T:

$$V' = -0.06426 + 2\cdot(0.0085043)T - 3(0.0000679)T^2.$$

Now set $V' = 0$ and solve for T:

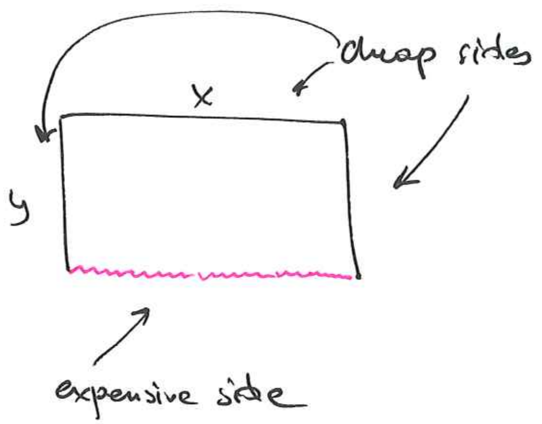
(Ben, you can tell them that they can either use some technology to find these roots of by graphing V' and use 2^{nd} - calc function on their calculators)

Anyway:

$T_1 = 3.96651$ C

$T_2 = 79.5318$ C ← outside of our range since $0 < T < 30$. X

(4)



Area $xy = 200$

so $y = \frac{200}{x}$. $0 < x < \infty$

$$\text{Cost of enclosure} = \underbrace{4\cdot(2y+x)}_{\text{cheaper side}} + \underbrace{12\cdot x}_{\text{expensive side}} = C(x,y)$$

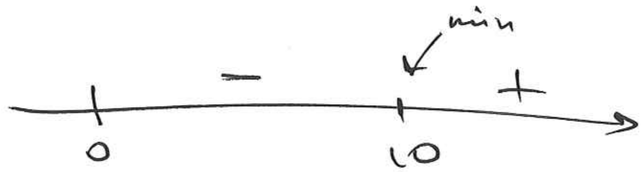
$$C(x) = 4\left(2\cdot\frac{200}{x}+x\right)+12x = \frac{1{,}600}{x} + 16x$$

Now need to minimize $C(x)$.

$$C'(x) = -\frac{1{,}600}{x^2} + 16 = \frac{16x^2 - 1{,}600}{x^2} = \frac{16(x^2-100)}{x^2}$$

$C'(x) = 0 \iff x^2 - 100 = 0$ so either $x = 10$

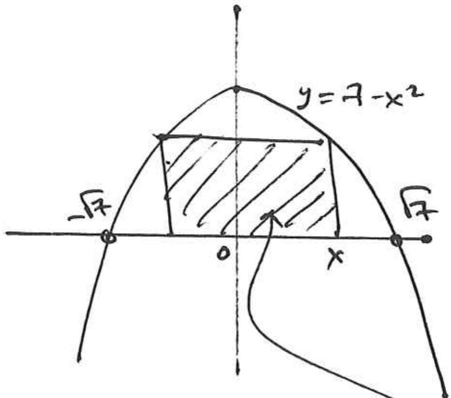
$\cancel{x = -10}$ outside of x domain



Thus C has local min @ $x = 10$, which actually global minimum, thus $y = \frac{200}{10} = 20$.

Dimensions: 10×20

(5)



$0 < x < \sqrt{7}$

Area of rectangle $= 2x \cdot y = 2x(7 - x^2)$

$= 14x - 2x^3 = A(x).$

Maximize area, thus

$A'(x) = 14 - 6x^2$

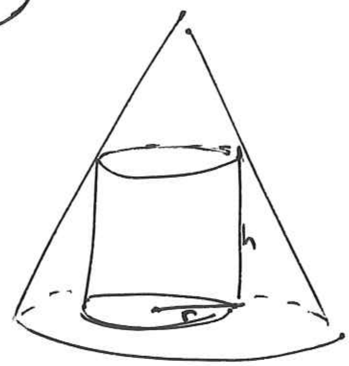
$A'(x) = 0 \iff 14 - 6x^2 = 0 \iff x = \pm\sqrt{7/3}$

negative out of our range, thus

$x = \sqrt{7/3}$ width $= 2x = 2\sqrt{7/3} \simeq 3.06$

heigh $= y(\sqrt{7/3}) = 7 - 7/3 = 14/3$

(6)



r – radius of the cylinder

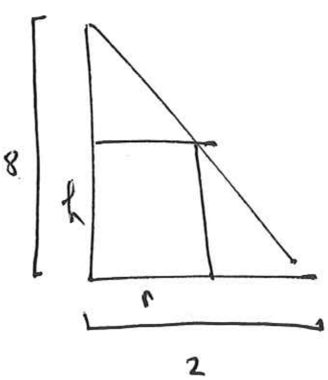
h – hieght of the cylinder

$0 \leq r \leq 2$.

Need to maximize $V = \pi r^2 \cdot h$.

Now we need to eliminate h from the above expression.

Use similar triangles:



$$\frac{8}{2} = \frac{8-h}{r} \quad \text{or} \quad \frac{8}{2} = \frac{h}{2-r}$$

$$4r = 8 - h$$

$$h = 8 - 4r$$

Thus: $V = \pi r^2 (8 - 4r) = 8\pi r^2 - 4\pi r^3$

Since $V(r)$ is continuous on the closed (Ben, don't use the term compact) $[0, 2]$ by the EVT (extreme value thm), $V(r)$ has a global max and it's attained either at the end points or critical points.

$V(0) = 0$

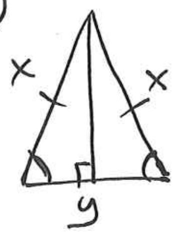
$V(2) = 0$

$V'(r) = 16\pi r - 12\pi r^2 = 4\pi r(4 - 3r) = 0 \iff r = 4/3$

$V(4/3) = 14.89347$ units^3 ← global max.

$r = 4/3$ $\quad h = 8 - 4 \cdot 4/3 = \frac{24 - 16}{3} = 8/3$

(7) - go slow on this one



$$P = 2x + y \quad \text{- perimeter} \quad \rightsquigarrow y = P - 2x$$

P - constant

$$A = \frac{1}{2} \cdot y \cdot \sqrt{x^2 - \left(\frac{y}{2}\right)^2}$$

$$= \frac{1}{4} y \sqrt{4x^2 - y^2}$$

$$= \frac{1}{4}(P-2x)\sqrt{4x^2 - (P-2x)^2}$$

$$= \frac{1}{4}(P-2x)\sqrt{\cancel{4x^2} - (P^2 - 4xP + \cancel{4x^2})}$$

$$= \frac{1}{4}(P-2x)\sqrt{4xP - P^2} = A(x)$$

$$A'(x) = \frac{1}{4}(-2)\sqrt{4xP-P^2} + \frac{1}{\cancel{4}}(P-2x) \cdot \frac{1}{2\sqrt{4xP-P^2}} \cdot 4P$$

$$= \frac{(P-2x)P}{2\sqrt{4xP-P^2}} - \frac{1}{2}\sqrt{4xP-P^2}$$

$$= \frac{(P-2x)P - (4xP-P^2)}{2\sqrt{4xP-P^2}} = \frac{P^2 - 2xP - 4xP + P^2}{2\sqrt{4xP-P^2}}$$

$$= \frac{P^2 - 3xP}{\sqrt{4xP-P^2}}$$

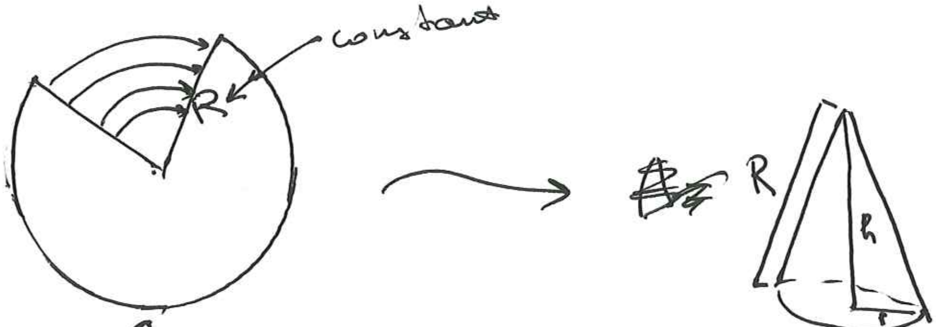
$$A'(x) = 0 \iff P^2 - 3xP = 0$$
$$\iff P = 3x$$
$$\iff x = P/3$$

$$A'(x) \text{ DNE} \iff 4xP - P^2 \leq 0$$
$$\iff 4x \leq P$$
$$\iff x \leq P/4.$$

8 Solution on the WebWork

just click : $\boxed{\text{solution}}$

(9)



~~Area~~

$$V = \frac{1}{3}\pi r^2 h =$$

h – cone height $(h>0)$

r – cone radius $(r>0)$

R – radius of the original piece of paper. $(R>0)$

By Pythagoras thm,

$$h^2+r^2=R^2 \rightsquigarrow r^2=R^2-h^2$$

Thus

$$V=\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\cdot(R^2-h^2)h = \frac{1}{3}\pi(R^2h-h^3) = V(h).$$

$$V' = \frac{1}{3}\pi(R^2-3h^2)$$

$$V'=0 \iff R^2-3h^2=0 \iff h=\pm\sqrt{\frac{R^2}{3}}$$

only + make sense thus $h = R/\sqrt{3}$.

$$\begin{array}{ccccc} & + & \overset{\swarrow \text{max}}{} & - & \\ \hline 0 & & R/\sqrt{3} & & \to V' \end{array}$$

$$V(R/\sqrt{3}) = \frac{1}{3}\pi\left(R^2\cdot R/\sqrt{3} - \frac{R^3}{3\sqrt{3}}\right)$$

$$= \frac{1}{3}\pi\left(\frac{2R^3}{3\sqrt{3}}\right) = \boxed{\frac{2\pi R^3}{9\sqrt{3}}}$$

(10) $E(v) = a v^3 \cdot \frac{L}{v-u}$

$$E'(v) = a\,3v^2 \cdot \frac{L}{v-u} + a v^3 \left(-\frac{L}{(v-u)^2}\right)$$

$$= \frac{a3v^2 L(v-u) - av^3 L}{(v-u)^2}$$

$$E'(v) = 0 \iff a3v^2 \cancel{L}(v-u) - \cancel{a}v^3\cancel{L} = 0$$

$$\iff 3v^2(v-u) - v^3 = 0$$

$$\cancel{\iff v^3(3v-3u-1)=0}$$

$$\cancel{v=0} \qquad \cancel{v=\frac{3u+1}{3}}$$

$$\iff v^2(3v-3u-v)=0$$

$$\iff \cancel{v=0} \quad \text{or} \quad 2v-3u=0$$

doesn't make sense in the problem.

$$v = \frac{3u}{2}$$

(11)

$$S = 6sh - \tfrac{3}{2}s^2\cot\theta + \frac{3\sqrt{3}}{2}s^2\csc\theta \quad , s, h \text{ - const.}$$

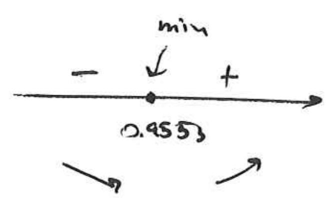
a)
$$\frac{dS}{d\theta} = -\tfrac{3}{2}s^2\frac{d}{d\theta}\left(\frac{\cos\theta}{\sin\theta}\right) + \frac{3\sqrt{3}}{2}s^2\cdot\frac{d}{d\theta}\left(\frac{1}{\sin\theta}\right)$$

$$= -\frac{3}{2}s^2\left(\frac{(-\sin\theta)\cdot\sin\theta - \cos\theta\cos\theta}{\sin^2\theta}\right) + \frac{3\sqrt{3}}{2}s^2\left(-\frac{\cos\theta}{\sin^2\theta}\right)$$

$$= -\frac{3}{2}s^2\left(\frac{-1}{\sin^2\theta}\right) \cancel{+} - \frac{3\sqrt{3}}{2}s^2\left(\frac{\cos\theta}{\sin^2\theta}\right)$$

$$= \frac{3s^2(1-\sqrt{3}\cos\theta)}{\sin^2\theta}$$

b)
$$\frac{dS}{d\theta} = 0 \iff 1-\sqrt{3}\cos\theta = 0 \iff \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 0.9553$$



c) Note
$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-1/3} = \sqrt{2/3}$$

$$S(\theta = 0.9553) = 6sh - \tfrac{3}{2}s^2\frac{1/\sqrt{3}}{\sqrt{2/3}} + \frac{3\sqrt{3}}{2}s^2\cdot\frac{1}{\sqrt{2/3}}$$

$$= 6sh - \frac{3}{2}s^2\frac{1}{\sqrt{2}} + \frac{3\sqrt{3}}{2}s^2\cdot\frac{\sqrt{3}}{\sqrt{2}}$$

$$= 6sh - \frac{3}{2}s^2\frac{1}{\sqrt{2}} + \frac{9s^2}{2\sqrt{2}}$$

$$= 6sh + \frac{6s^2}{2\sqrt{2}}$$

(12) $P(x) = Kx\left(\frac{r-x}{r}\right) = Kx - \frac{K}{r}x^2$

$$P'(x) = K - \frac{K}{r}\cdot 2x = 0 \iff x = \frac{r}{2}$$

sign of $P'(x)$:

$$\begin{array}{ccccl} + & & \text{max} & & - \\ \hline & & r/2 & & x \;\; P' \end{array}$$

Thus @ $x = r/2 \rightarrow$ local and global max.

$$F_e(r/2) = K\frac{(r - r/2)}{r} = K\frac{r/2}{r} = \frac{K}{2}$$

Of course the maximum population in this model would occur when there is no harvesting: $x = 0$.

In this case:

$\frac{dF}{dt} = rF(1 - F/K)$ is the logistic growth model. The max occurs when we reach the carrying capacity K.

(13) $$P(c) = \frac{1{,}000c}{1+2{,}025c^2}$$

$$P'(c) = \frac{1{,}000(1+2{,}025c^2) - 1{,}000c\cdot 4{,}050c}{(1+2{,}025c^2)^2}$$

$$= \frac{1{,}000 + 1{,}000\cdot 2{,}025c^2 - 1{,}000\cdot 2{,}025\cdot 2c^2}{(1+2{,}025c^2)^2}$$

$$= \frac{1{,}000 - 1{,}000\cdot 2{,}025c^2}{(1+2{,}025c^2)^2}$$

$$P'(c) = 0 \iff 1{,}000 - 1{,}000\cdot 2{,}025c^2 = 0$$

$$\iff c = \sqrt{\frac{1}{2{,}025}}$$ ($c = -\sqrt{1/2025}$ doesn't make any sense in this problem)

$$P\left(\sqrt{\frac{1}{2{,}025}}\right) = \frac{1000\cdot\sqrt{1/2{,}025}}{1+2{,}025\cdot\frac{1}{2{,}025}}$$

$$= \frac{500}{\sqrt{2{,}025}}$$

14

$$r(x) = \frac{\ln(L(x)M(x))}{x} \qquad 0 < x < \infty.$$

$$= \frac{\ln\left(e^{-0.21x} \cdot 6x^{0.5}\right)}{x}$$

$$= \frac{\ln\left(e^{-0.21x}\right) + \ln\left(6x^{0.5}\right)}{x}$$

$$= \frac{-0.21 \cdot x + \ln 6 + 0.5 \ln x}{x} = -0.21 + \frac{\ln 6 + 0.5 \ln x}{x}$$

$$r'(x) = \frac{\frac{1}{2} \cdot \frac{1}{x} \cdot x - \left(\ln 6 + \frac{1}{2}\ln x\right) \cdot 1}{x^2} = \frac{\frac{1}{2} - \ln 6 - \frac{1}{2}\ln x}{x^2}$$

$$r'(x) = 0 \iff \frac{1}{2} - \ln 6 - \frac{1}{2}\ln x = 0$$

$$\iff \ln x = 1 - 2\ln 6$$

$$\iff x = e^{1 - 2\ln 6} = e\, e^{\ln 6^{-2}} = \frac{e}{36} \approx 0.0755$$

r'

	$+$	max	$-$
0		$e/36$	