Quiz #7

Directions: Carefully read each question below and answer to the best of your ability in the space provided. You **MUST** show your work to receive full credit!

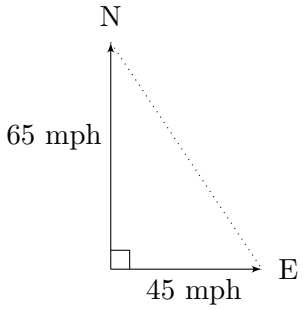
1. (5 points) Find $\dfrac{dy}{dx}$ of $xy^2 + 2y + 3x = 4$.

Solution:

$$\begin{aligned}
\frac{d}{dx}(xy^2 + 2y + 3x) &= \frac{d}{dx}(4) \\
y^2 + x \cdot 2y \cdot \frac{dy}{dx} + 2\frac{dy}{dx} + 3 &= 0 \\
2xy\frac{dy}{dx} + 2\frac{dy}{dx} &= -3 - y^2 \\
\frac{dy}{dx}(2xy + 2) &= -(3 + y^2) \\
\frac{dy}{dx} &= \boxed{-\frac{3 + y^2}{2xy + 2}}
\end{aligned}$$

2. (5 points) Two trains leave a station at 1:00 PM. One travels north at 65 mph and the other travels east at 45 mph. How fast is the distance between the two trains changing at 4:00 PM? (**Hint:** Draw a picture)

Solution:



Let's call distance between trains D, x - distance that East train covered, and y - distance that North train covered, then using Pythagorean Theorem, we get

$$D^2 = y^2 + x^2.$$

Since we are interested in finding how fast D is changing, then we will take derivative with respect to t, time

$$2D\frac{dD}{dt} = 2y\frac{dy}{dt} + 2x\frac{dx}{dt}.$$

We know that $\dfrac{dy}{dt} = 65$ mph and $\frac{dx}{dt} = 45$ mph, but we need to find D, x and y exactly at 4:00 PM. Since trains left a station at 1:00 PM, then at 4:00 PM, that would be 3 hours. Therefore, east train traveled $3 \cdot 45 = 135$ miles, north - $3 \cdot 65 = 195$ miles and $D = \sqrt{195^2 + 135^2} \approx 237.171$ miles. Thus,

$$\sqrt{195^2 + 135^2}\frac{dD}{dt} = 195 \cdot 65 + 135 \cdot 45$$

and

$$\frac{dD}{dt} = \frac{195 \cdot 65 + 135 \cdot 45}{\sqrt{195^2 + 135^2}} \approx \boxed{79.057 \text{ mph}}.$$

Name: _____

Section (circle one): 003 004

Question:	1	2	Total
Points:	5	5	10
Score:			